

Field visualisation as an aid in teaching electromagnetic field theory

E. Kurgan

AGH University of Science and Technology
Kraków, Poland

ABSTRACT: The visualisation procedure of three-dimensional graphical objects is not simple for at least two reasons. First, most *real world* problems are three-dimensional by nature, but such problems have to be displayed on two-dimensional screens. Thus the 3D viewing process is inherently more complex than 2D visualisation. The solution is to use geometrical projection, which projects 3D objects onto a 2D plane yielding a two-dimensional image. But these images can lack depth. The perception of depth in humans arises in various ways, including through perspective, colour, shadow and changes in size. Visualising in three dimensions draws on ways to perceive depth. For example, each eye in a human has a slightly different view of distant objects and the angular difference varies with distance. When both eyes focus on an object, they produce a stereoscopic view.

INTRODUCTION

To visualise a drawing or the result of a calculation simple graphical actions are applied, such as shifting, scaling, rotating or clipping. In each case, the resulting image is properly defined. We have only to specify a window on the 2D world coordinates and a viewport on the 2D viewing surface (for example, a computer monitor). Conceptually, the *real* drawings in the 2D world are clipped against the window and then transformed into the viewport for display. All who begin to write their own electromagnetic field simulation program must first answer some fundamental questions, e.g. how to create parts that can be used to build assemblies; how to maintain them; and the format to store geometric models on disk. As one example, these parts and assemblies may be used as geometry for finite or boundary element models.

Other important problems are: How to introduce material properties; how to mesh a given model for finite and/or boundary elements; in which way to organise the results of viewing analysis; and so on. Many try to do all or most of this work by themselves and from the beginning. This mostly results in serious consequences. The time needed to write such field simulation programs is much more than is necessary, and the geometry models together with computational results are not transferable to another simulation or visualisation programs. This last condition is especially important for programs written for use as commercial products. Such self-written pre- and post-processors mostly have much lower capabilities and contain a substantial number of bugs.

This is caused by the fact that a great deal of time and money is needed to write and carefully test such programs. The extent of the work is comparable to writing - or even exceeds the time needed to write - the main electromagnetic field analysis program. Moreover, the people for whom a main scientific area of interest is electromagnetic, have no essential knowledge of how to write this kind of software. In most cases they concentrate on graphical input and output techniques instead of on the solutions to field problems.

The solution to these problems is to use existing commercial pre- and post-processor programs. There is available a good standardised geometric modelling technique that uses industry-standard file formats (for example, IGES, SAT, STEP) and can carry out FEM meshing. The visualisation parts of these programs are mostly easy to use and work in an intuitive way [1][2]. They make the building and testing of software prototypes in such a way that it reduces the number of costly physical prototypes needed. The time and expense needed to learn and effectively use such products is refunded very quickly.

Successful application of the finite element method to solving electromagnetic field problems highly depends not only on the systematic way that formulation of the field equations, discretisation and solution of the resulting algebraic equations is achieved but also on the way post-processing techniques are applied. Methods such as two- and three-dimensional scalar and vector field visualisation are powerful tools not only for scientific research but also for teaching students the principles of electromagnetic theory and application of the finite element method to solving nontrivial field problems. The student can view the field from different viewpoints or observe the influence of changes in geometry and material parameters on field shape. It is advisable to teach the students in such a way that these facts are fully preserved. This gives the students some intuitive ideas on concepts, principles and theorems that otherwise would be difficult to explain and to understand. As a result, learning becomes more attractive and effective.

To achieve better educational effects, the visualisation must be interactive, so that the user could in real time introduce changes in geometry and material properties. Contrary to two-dimensional problems, visualisation of 3D scalar and vector fields is a challenging task, because both the magnitude and direction of the vector at each point must be displayed. The difficulty comes from the fact that it is impossible to show all details contained in a 3D space on a 2D plane.

The idea is to use time as the third dimension in addition to the 2D image plane. A typical way to realise this idea is through animation. An interactive display allows the student to influence post-processing by moving the observation point and the scene to observe the electromagnetic field in various ways. Other common methods to represent 3D fields on the plane are equipotential lines and surfaces and flux lines.

The purpose of this paper is twofold: to give an overview of the concept of visualisation for teaching purposes and to present some simple but nontrivial methods of drawing equipotential and field lines, which could be implemented on a computer in a typical student electromagnetic project.

FUNDAMENTAL DESIGN CONCEPTS

Pre- and post-processors make it possible to:

- develop initial design concepts;
- exercise software prototypes through design analysis to determine if a model meets design criteria;
- modify a design according to the results of field analysis;
- describe design work with text and dimensional information;
- mesh a volume or boundary;
- introduce boundary and initial conditions;
- visualise computational results.

The only task the user must fulfil is to write an adequate procedure that must couple the analysis program with the pre- and post-processor. Generally, it is not an easy problem. All geometric objects are organised in a design hierarchy, so as to organise and establish relationships among the parts of a design model. The design hierarchy consists of parts, assemblies and envelopes collectively called design objects (see Figure 1). The design objects allow implementation of a bottom-up or top-down approach to problems in electromagnetic field computation.

When creating a part or assembly, one can usually work with great conceptual freedom. This freedom of design is preserved when working with a design hierarchy, as summarised in the following list:

- One can add, modify and delete design objects at any time.
- The design work can begin at any level of the design hierarchy. A project can start by creating a set of assembliesand individual parts, and, if desired, assigning them to assemblies at some later time.
- Design objects can be organised in any way that is logical and useful.
- Any portion of an existing hierarchy can be used for different projects by instantiating or copying.
- To accomplish these tasks one can use adequate design tools that help to optimise the design effort.

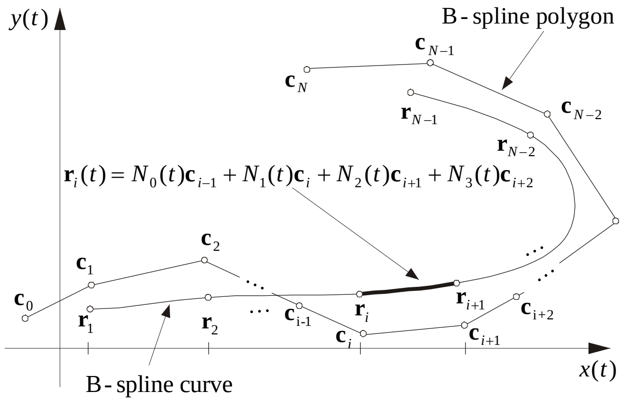

Figure 1: B-spline and approximated curve.

The most versatile construction curve spline $\mathbf{r}(t)$ is defined as a map of a collection of sections $i \le t \le i + 1$ onto $\mathbf{E}^2$, where each interval is mapped onto a polynomial segment [1]

$$\mathbf{r}_i(t) = N_0(t)\mathbf{c}_{i-1} + N_1(t)\mathbf{c}_i + N_2(t)\mathbf{c}_{i+1} + N_3(t)\mathbf{c}_{i+2} \tag{1}$$

where $N_0(t)$, $N_1(t)$, $N_2(t)$, $N_3(t)$ are B-spline shape functions and $\mathbf{c}_{i-1}$, $\mathbf{c}_i$, $\mathbf{c}_{i+1}$, $\mathbf{c}_{i+2}$ are control points for this spline segment (Figure 1).

The collection of points $t = i$, $i = 1, 2, \dots, N - 1$, is called a parameter knot sequence and a set of points $\mathbf{c}_i$, $i = 0,1, \dots, N$, is named ‘control points’. The four basic B-spline functions are given by:

$$N_0(t) = \frac{1}{6}(1-t)^3 \tag{2}$$

$$N_1(t) = \frac{1}{2}t^3 - t^2 + \frac{2}{3} \tag{3}$$

$$N_2(t) = \frac{1}{2}t^3 + \frac{1}{2}t^2 + \frac{1}{2}t + \frac{1}{6} \tag{4}$$

$$N_3(t) = \frac{1}{6}t^3 \tag{5}$$

and are shown in Figure 2. The whole B-spline curve can be given by the following representation:

$$\mathbf{r}(t) = \sum_{i=0}^{3} N_i\left(t - \lfloor t \rfloor\right)\mathbf{c}_{\lfloor t \rfloor + i} \tag{6}$$

where, symbol t denotes an integer part of the parameter t [2].

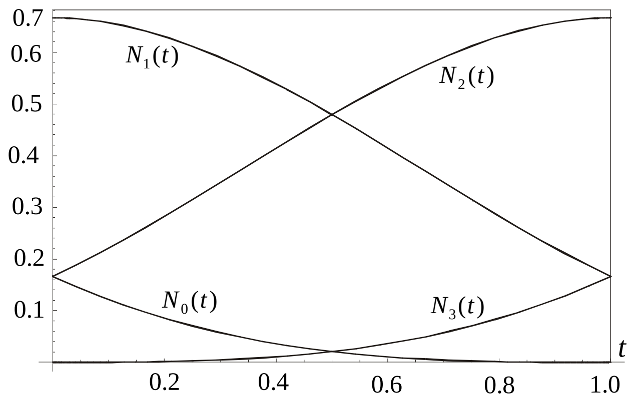

Figure 2: Shape curves for B-spline approximations.

FLUX LINE REPRESENTATION

Equipotential lines and surfaces and flux lines are the standard techniques for visualising constant vector fields. The difficulty arises from the fact that both the magnitude and the direction of the vector at each co-ordinate point have to be displayed. To represent these data contained in 3D space completely in 2D space is impossible. The flux line representation is a convenient, practical and very popular way to represent vector fields in electromagnetic applications on a 2D plane.

A flux line is a line everywhere tangent to the vector field. In other words, the tangent direction of any point along a flux line coincides with the vector at that point. Also, the density of flux lines is proportional to the density of the vector field at that point. The calculation of flux lines of, for example, the electrostatic strengths vector *E(r)* depends on integrating a following expression:

$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} = \mathbf{E}(\mathbf{r}) \tag{7}$$

where $r = (x, y, z)$ is a position in 3D space.

The integration variable t is not a time variable, because the integration occurs only at one instant in time. If vector E has three components $E = (E_x, E_y, E_z)$, the equation (7) can be written in a more direct way as:

$$\mathrm{d}\,x = E_x(x, y, z)\,\mathrm{d}\,t \tag{8}$$

$$\mathrm{d}\,y = E_y(x, y, z)\,\mathrm{d}\,t \tag{9}$$

$$\mathrm{d}\,z = E_z(x, y, z)\,\mathrm{d}\,t \tag{10}$$

The idea is to integrate (x, y, z) using (dx, dy, dz) until the boundary of the problem is reached or the flux line closes. The algorithm can be broken into two parts: the first, where integration of (8), (9) and (10) is performed; and the second, which is responsible for evaluating the vector function E at previously calculated point r in space. The first step is relatively straightforward. Assuming that we know the vector field $E = (E_x, E_y, E_z)$, then equations (8), (9) and (10) are integrated numerically by, for example, a fourth-order Runge-Kutta formula.

As an example, let us consider the computation of the potential in electrochemical corrosion. After solving this problem with Maxwell's equation, we can draw equipotential lines and vector fields as shown in Figures 4 and 5.

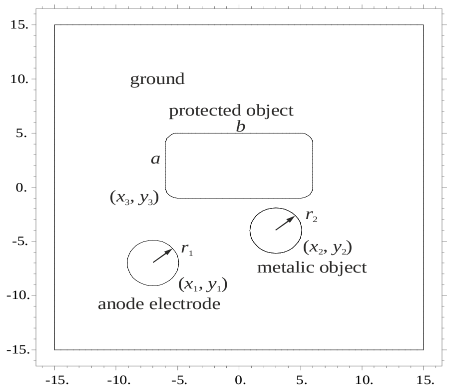

Figure 3: Protection of underground object by cathodic protection.

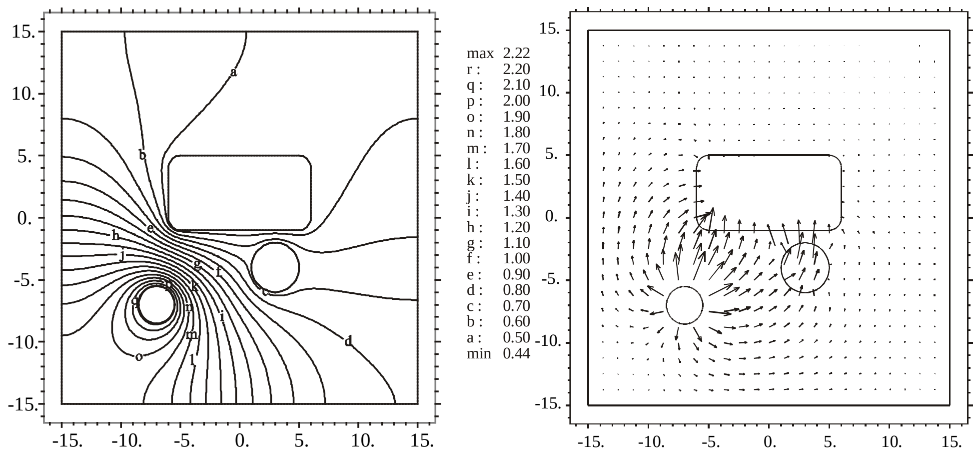

Figure 4: Equipotential lines and electric field in a computational domain.

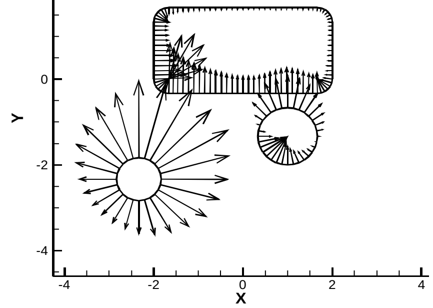

Figure 5: Electric field is shown in the form of vectors on problem boundaries.

CONCLUSIONS

Graphical representation of different variables in electromagnetic field computation can be a highly useful tool in electromagnetic field education. Students can verify their numerical results and compare them with expectations. They can prove if the boundary conditions are fulfilled and if the field has the distribution that accords with theory.

REFERENCES

1. Adrey, R.A. and Niku S.M., Computer modeling of corrosion using the Boundary Element Method. *Computer Modeling in Corrosion*, STP-1154, Philadelphia, 248-264 (1992).
2. Munn, R.S., *Microcomputer Corrosion Analysis for Structures in Inhomogeneous Electrolytes*. In: Heidersbach, F.J. and Erbar, R. (Eds), Corrosion/86 Symposium on Computers in Corrosion Control, NACE, Houston, TX, 240-255 (1986).