The use of mathematical programs and numerical methods in teaching selected topics in circuit theory based on Maple and MATLAB

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ABSTRACT: This article addresses the challenge of developing techniques for the effective teaching of electric circuit theory. It describes the examples and experience of using mathematical programs to solve tasks in the computational theory of electrical circuits at the AGH University of Science and Technology in Kraków, and the State Higher Professional School in Tarnów, Poland. The method of deriving loop matrix impedances and the loop voltage vector in electric linear circuits, containing dependent sources, is described in this article. The transformation equations of the electric circuit were derived within the Maple mathematics software program. This article also presents two examples of the use of numerical methods. The first example shows the calculation of a complex three-phase circuit. The second example presents an analysis of an electrical circuit containing an element with nonlinear current-voltage characteristics, such as a tunnel diode. These examples concentrate on the numerical capabilities of the computer program MATLAB for solving linear and nonlinear equations and its plotting capabilities. Through these abilities, more time can be devoted to the analysis of phenomena that take place, while problems including electric circuits are being solved. Therefore, the authors perceive mathematical programs as a very useful didactical tool in teaching electric circuit theory.

Keywords: Electric circuit theory, network analysis, graph theory, numerical methods, Maple, MATLAB

INTRODUCTION

Electric circuit theory plays an important role in the understanding of electric and magnetic phenomena and it allows students to understand the principles of electrical devices. Many branches of electrical engineering, such as electronics, power, electric machines, communications, control and instrumentation are based on electric circuit theory [1][2]. The goal of circuit theory is to make quantitative and qualitative analyses of the electrical behaviour of circuits [3]. Therefore, a fundamental electric circuit theory course is very important for electrical engineering students in the first stage of their education, as well as students specialising in other disciplines of the physical and technical sciences. This is because circuits are an excellent model for the study of energy and power systems in general and also because of the applied mathematics, physics and topology involved.

The most typical forms of teaching of circuit theory at the university level are lectures, classes, laboratory sessions and projects. Classes with students are conducted in order to enhance and supplement the knowledge obtained during lectures. Exercises aim to teach students how to solve tasks and problems of electric circuits. Exercises are associated with solving computational tasks of circuit theory.

Generally, there are three stages in this process: formulating the circuit equations depending on the chosen method of analysis, solving equations of the circuit and discussing the results. The first stage requires the knowledge of theory and the methods of circuit analysis. The time for its realisation depends on the complexity of the problem being solved. The final stage enables students to understand the phenomena occurring in electric circuits and the way electrical systems operate. This is possible due to the analysis and discussion of the results. The second stage is very time-consuming. It takes over 80% of the total time spent on the task. During mathematical computations, students very often make mistakes. This leads to incorrect results and makes the proper execution of the third stage impossible. Therefore, the application of computers in the second stage reduces the amount of time spent by students in performing computational tasks and eliminates calculation errors. Nowadays, computers allow for relatively quick calculations and the elimination of calculation errors [4-6].

This article describes examples of using the mathematical programs to solve tasks in computational theory of electrical circuits at the AGH University of Science and Technology in Kraków and the State Higher Professional School in Tarnów, Poland [7]. The article presents formulating structural matrices of equations of electric circuits containing dependent sources, with the use of the Maple program and examples of the use of numerical methods with MATLAB.

In next sections of the article, a method for the analysis of electric circuits based on loop transformation with example, as well as two typical problems, is presented and discussed, with the use of numerical methods that could be used as illustrative examples, in the exercise or homework. The first example, dealing with use of the numerical method, shows the calculation of a complex three-phase circuit. The next example presents an analysis of the electrical circuit containing an element with non-linear current-voltage characteristics, such as tunnel diodes.

NETWORK ANALYSIS - LOOP TRANSFORMATIONS AND LOOP SYSTEM OF EQUATIONS

The goal of network analysis is to determine the voltages and currents associated with the elements of an electrical network so as to predict the electrical behaviour of real physical circuits. The purpose of these predictions is to improve their design; particularly, to decrease their cost and improve their performance under all conditions of operation. Kirchhoff's Voltage Law (KVL) and Kirchhoff's Current Law (KCL) are fundamental postulates of circuit theory. They are valid regardless of the nature of circuit elements. Therefore, the separation between the equations of Kirchhoff's Laws and that of elements' characteristics is natural.

Kirchhoff's equations are prescribed by the topology of the circuit, that is, the way circuit elements are interconnected. The element characteristic is prescribed by the voltage-current (v-i) relations given by the laws of physics. However, these equations involve a large quantity of variables. The theory of graphs plays a fundamental role in exploring the structural properties of electrical circuits [8]. In circuit analysis, using graph theory, there is compliance with three transformations: loop transformations, cut-set transformations and node transformations. These transformations diminish the number of unknowns. Only the method of analysis of electric circuits based on loop transformation is applied in this article. Given solutions can be applied in the manner of methods of analysis based on extant transformations.

An electrical circuit is an interconnection of electrical network elements, such as resistances, capacitances, inductances, independent and dependent voltages and current sources. Each network element is associated with two variables: the voltage variable $v(t)$ and the current variable $i(t)$. The electrical network N can be represented by corresponding with its directed graph G. Let T be a spanning tree of an electrical network. Let I_c be the column vectors of chord currents and I_b be the column vectors of branch currents with respect to T. The sub-matrix of circuit matrix $\mathbf{B}_c$ corresponding to the fundamental circuits defined by the chords of a spanning tree T is called fundamental circuit matrix $\mathbf{B}_f$ of G with respect to the spanning tree T. The equation below presents the loop transformation:

$$I_b = \mathbf{B}_f^T I_c \tag{1}$$

As can be seen from the loop transformations, not all these variables are independent. Furthermore, in place of Kirchhoff's Voltage Law equations, the loop transformation, which involves only chord currents as variables, can be used. Kirchhoff's Voltage Law is given below:

$$\mathbf{B}_f V_b = 0 \tag{2}$$

Assuming that $\mathbf{Z}_N$ is the branch impedance matrix, the voltage-current relations for every branch are:

$$V_b = \mathbf{Z}_N I_b \tag{3}$$

The advantage of these transformations is that it enables different systems of network equations known as loop systems to be established. In deriving the loop system, the loop transformation is used, and loop variables (chord currents) serve as independent variables. Partition of the chord currents vector I_c and element voltage vector V_b is:

$$I_c = \begin{bmatrix} I_{c\_nsc} \\ J \end{bmatrix} \quad \text{and} \quad V_b = \begin{bmatrix} V_{b\_nsv} \\ E \end{bmatrix} \tag{4}$$

where I_{c_nsc} is the vector of currents in the non-current source chords of G, J is the current sources vector, V_{b_nsv} is the vector of branches voltage in the non-voltage source and E is the voltage sources vector.

Using the above dependencies, this equation is derived:

$$\mathbf{B}_f \mathbf{Z}_N \mathbf{B}_f^T I_{c\_ns} = \mathbf{B}_f \left(E - \mathbf{Z}_N J \right) \tag{5}$$

Equation (5) is called the loop system of equations and matrix $\mathbf{B}_f \mathbf{Z}_N \mathbf{B}_f^T$ is the loop impedance matrix.

Since presenting examples is a fundamental element of the didactic process, a computational example is introduced below that illustrates the problems above. The formulation of structural matrices of the electric circuit, with the help of the Maple software program, is given below.

ILLUSTRATIVE EXAMPLE WITH DEPENDENT SOURCE

Since there are various graphical signs, the elements of electrical circuit in Figure 1 show the symbols used in this article.
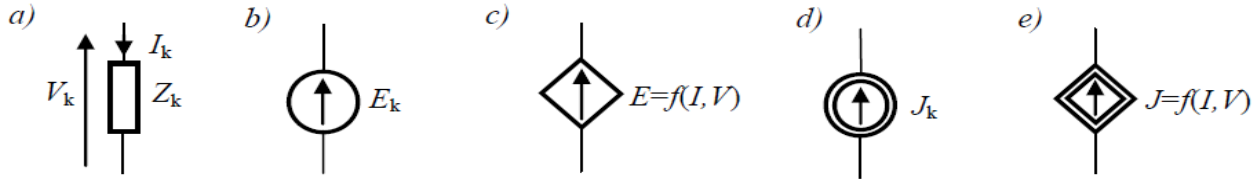

Figure 1: a) Impedance Z_k with current I_k and voltage V_k, b) Independent voltage source, c) Dependent voltage source, d) Independent current source, e) Dependent current source.

Now, consider linear circuit impedances, as well as independent and dependent voltage source and current sources. The schema of the electrical network N is shown in Figure 2a. Graph G representation of N corresponding to branch circuit with the spanning tree is shown in Figure 2b.
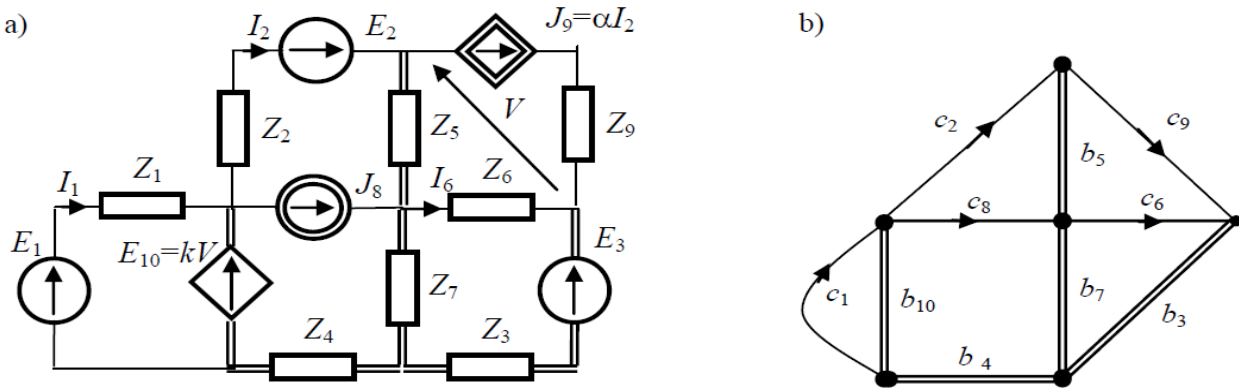

Figure 2: a) The electrical network N with a spanning tree and directed co-spanning tree; b) Graph G representation of N.

Consider also the connected graph and its spanning tree. A sub-graph of G is a spanning tree of G, if the sub-graph is a tree and contains all the vertices of G. The spanning tree of the network of Figure 2a is shown in Figure 2b. The edges of a spanning tree T are called the branches of T. The branches of the tree are marked with double lines. For the given spanning tree of connected graph G, the co-spanning tree relative to T is the sub-graph of G induced by the edges that are not present in T. The edges of a co-spanning tree are called chords. The co-spanning tree relative to the spanning tree T of Figure 2b consists of these chords: c_1, c_2, c_6, c_8, c_9. The spanning tree T was chosen so as to make voltages and currents, which control dependent voltage sources and current to be in chords.

Because only *current* chords are of interest to the method discussed here, only those signals were marked. The chord currents vector Ic is as follows:

$$I_c = \begin{bmatrix} I_{c\_nsc} \\ J \end{bmatrix} = [I_1\ I_2\ I_3\ J_8\ J_9\ ]^T \tag{6}$$

Wanted unknown chord currents are:

$$I_{c\_nsc} = \begin{bmatrix} I_1 & I_2 & I_6 \end{bmatrix}^T \tag{7}$$

The voltages and currents, which control dependent sources are expressed respectively by means of chord currents and independent voltages and *current* sources. Because the currents of the branches of the spanning tree are linear, the equation there is:

$$V = Z_5(I_2 - \alpha I_2) + Z_6 I_6 \tag{8}$$

Shown below is how Maple is used to perform mathematical transformations. The `linalg` packages (Maple library) contain functions that help users to work with matrix and linear system equations. An equation can be created in Maple by any method, which leads to the construction of algebraic (or any other) equations [9][10].

For controlled sources, current-voltage dependence is written as:

```
> restart;with(linalg):
> E[10]:=k*V;
> J[9]:=alpha*Ic[2];
> V:=Z[5]*(Ic[2]-J[9])-Z[6]*Ic[6];
```

Kirchhoff's Voltage Law equation around the loop with chord c_1 is written as:

```
> KVL1:=Z[1]*Ic[1] = E[1]-k*V;
```

Kirchhoff's Voltage Law equation around the loop with chord c_2 is written as:

```
> KVL2:=(Z[2]+Z[4]+Z[5]+Z[7])*Ic[2]+Z[7]*Ic[6]+(Z[4]+Z[7])*J[8]-(Z[5]+Z[7])*J[9] =E2+k*V;
```

Kirchhoff's Voltage Law equation around the loop with chord c_6 is written as:

```
> KVL3:=Z[7]*Ic[2]+(Z[3]+Z[7]+Z[6])*Ic[6]-Z[7]*J[8]+(Z[3]+Z[7])*J[9] = -E[3];
```

Loop system of equations:

```
> set_eqn:= [KVL1,KVL2,KVL3]:
```

Determining loop impedance matrix $\mathbf{B}_f\mathbf{Z}_N\mathbf{B}_f^T$ and matrix $\mathbf{B}_f(E-\mathbf{Z}_N J)$ uses the following:

```
> BZBT:=genmatrix(set_eqn,[Ic[1],Ic[2],Ic[6]],'b');
```

$$BZBT := \begin{bmatrix} Z_1 & k\,Z_5\,(1-\alpha) & -k\,Z_6 \\ 0 & -(Z_5+Z_7)\,\alpha+Z_2+Z_4+Z_5+Z_7-k\,Z_5\,(1-\alpha) & Z_7+k\,Z_6 \\ 0 & (Z_3+Z_7)\,\alpha+Z_7 & Z_3+Z_7+Z_6 \end{bmatrix}$$

```
> B:=eval(b):BE_ZJ:=matrix(3,1,B);Icns:=matrix(3,1,[Ic[1],Ic[2],Ic[6]]);
```

$$BE\_ZJ := \begin{bmatrix} E_1 \\ E2-(Z_4+Z_7)\,J_8 \\ -E_3+Z_7\,J_8 \end{bmatrix} \qquad Icns := \begin{bmatrix} Ic_1 \\ Ic_2 \\ Ic_6 \end{bmatrix}$$

Then, the properties of the loop impedance matrix are explored, and the vector of currents I_{c_nsc} in the non-source chords is found. Remaining currents and voltages in the electric circuit can be calculated from Equations (1) and (3).

THREE-PHASE SYSTEMS

Generation, transmission and distribution of electricity is accomplished by a three-phase alternating currents system. A three-phase AC supply is carried by three conductors called lines. The currents in these conductors are known as line currents I_L. Sources of three-phase supplies, i.e. generators, are usually connected in star, whereas three-phase transformer windings, motors and other loads may be connected either in star or delta. Loads of three-phase systems may be balanced or unbalanced [2].

The first example is aimed at making students familiar with the basic knowledge of three-phase systems necessary for further research. The concepts of AC steady-state analysis are applied to analyse the circuit; concepts that are introduced during an earlier part of the circuit theory course. The analysis of circuits, containing such elements as a resistor R, inductor L, capacitor C and mutual inductance M, excited by sinusoidal three-phase sources E, is considerably simplified when phasor voltages V and phasor currents I are used (they are represented by capital letters respectively).

The analysis of a three-phase network involves determining each of the unknown branch currents and voltages. Consider a mixed delta and star connected unbalanced load, as shown in Figure 3. In this circuit, the balanced three-phase generator voltage $E = 230$ volts. Line impedances and the load of the star's impedances are respectively:

$Z_{L1} = Z_{L1} = Z_{L1} = 0.2 + j0.1\Omega$, $Z_N = 0.3 + j0.1\Omega$, $Z_1 = 7 + j4\Omega$, $Z_2 = 5 - j5\Omega$, $Z_3 = 5 - j5\Omega$.

Delta load impedances are, respectively: $Z_{12} = R_1 + jX_{L1}$, $Z_{23} = R_2 - jX_C$, $Z_{31} = R_3 + jX_{L3}$, where: $R_1 = 10\ \Omega$, $R_2 = 15\ \Omega$, $R_3 = 5\ \Omega$ and $X_{L1} = 8\Omega$, $X_C = 5\Omega$, $X_{L3} = 10\Omega$, $X_M = 3\Omega$.

Now, find all the line currents and load currents and draw a complete phasor diagram for the line currents, delta currents and star currents.
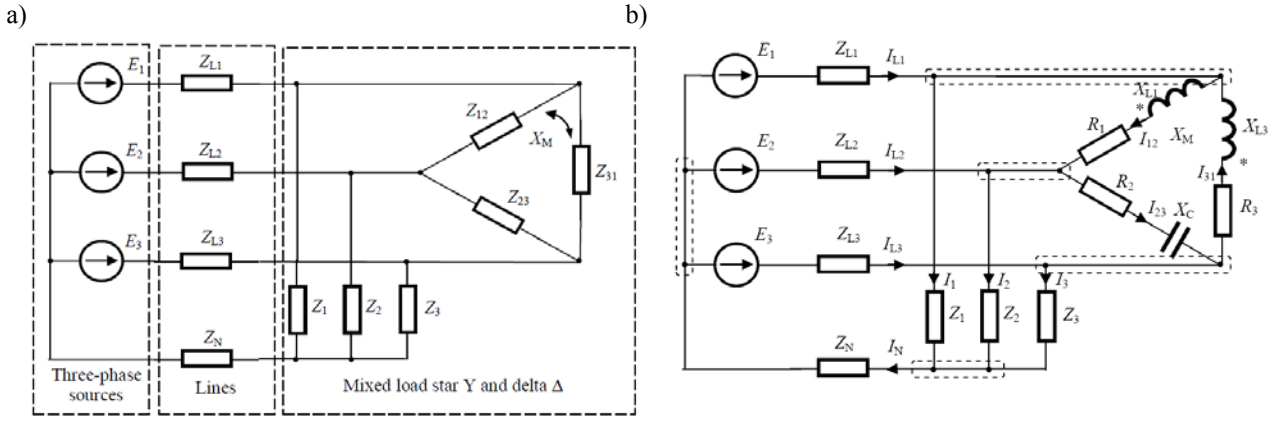
Figure 3: The three-phase circuit; a) Y-connected three-phase generator with a mixed delta and star connected unbalanced load; b) Circuit diagram with cut-set, currents in lines I_L and currents in load I_Y, I_Δ.

As shown in Figure 3a, for all the impedances in the circuit, a current variable with arbitrary polarity is assigned. The analysed circuit has $N = 5$ cut-set (super node), where N is the total number of the cut-set in the circuit. Now, write Kirchhoff's Current Law equations for an N-1 cut-set. The algebraic sum of currents at the super node is zero.

$$\begin{aligned}
I_{L1} - I_1 - I_{12} + I_{31} &= 0 \\
I_{L2} - I_2 + I_{12} - I_{23} &= 0 \\
I_{L3} - I_3 + I_{23} - I_{31} &= 0 \\
I_1 + I_2 + I_3 - I_N &= 0
\end{aligned} \tag{5}$$

The relationship between current phasor I and voltage phasor V on the branch impedance Z is as follows:

$$V = Z \cdot I \tag{6}$$

Expression (6) is referred to as Ohm's Law for AC circuits.

Applying Kirchhoff's voltage equations for B-N+1=6 independent loops, where B is the number of branches in the circuit, one obtains:

$$\begin{aligned}
E_1 - Z_{L1} \cdot I_{L1} - (R_1 + jX_{L1}) \cdot I_{12} - jX_M \cdot I_{31} + Z_{L2} \cdot I_{L2} - E_2 &= 0 \\
E_2 - Z_{L2} \cdot I_{L2} - (R_2 - jX_C) \cdot I_{23} + Z_{L3} \cdot I_{L3} - E_3 &= 0 \\
E_1 - Z_{L1} \cdot I_{L1} + (R_3 + jX_{L3}) \cdot I_{31} - jX_M \cdot I_{12} + Z_{L3} \cdot I_{L3} - E_3 &= 0 \\
E_1 - Z_{L1} \cdot I_{L1} - Z_1 \cdot I_1 - Z_N \cdot I_N &= 0 \\
E_2 - Z_{L2} \cdot I_{L2} - Z_2 \cdot I_2 - Z_N \cdot I_N &= 0 \\
E_3 - Z_{L3} \cdot I_{L3} - Z_3 \cdot I_3 - Z_N \cdot I_N &= 0
\end{aligned} \tag{7}$$

There are sufficient simultaneous linear equations to solve for unknown currents at branches $\mathbf{I_B}$. The above set of Equations (5) and (7) in matrix notation takes the following form:

$$\mathbf{Matrix} \cdot \mathbf{I_B} = \mathbf{Right\_Side} \tag{8}$$

$$
\begin{bmatrix}
1 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 1 \\
0 & 1 & 0 & 0 & 0 & -1 & 0 & 1 & -1 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 1 & -1 \\
0 & 0 & 0 & -1 & 1 & 1 & 1 & 0 & 0 & 0 \\
Z_{L1} & -Z_{L2} & 0 & 0 & 0 & 0 & 0 & Z_{12} & 0 & jX_M \\
0 & Z_{L2} & -Z_{L3} & 0 & 0 & 0 & 0 & 0 & Z_{23} & 0 \\
-Z_{L1} & 0 & Z_{L3} & 0 & 0 & 0 & 0 & -jX_M & 0 & Z_{31} \\
Z_{L1} & 0 & 0 & Z_N & Z_1 & 0 & 0 & 0 & 0 & 0 \\
0 & Z_{L2} & 0 & Z_N & 0 & Z_2 & 0 & 0 & 0 & 0 \\
0 & 0 & Z_{L3} & Z_N & 0 & 0 & Z_3 & 0 & 0 & 0
\end{bmatrix}
\cdot
\begin{bmatrix}
I_{L1} \\ I_{L2} \\ I_{L3} \\ I_N \\ I_1 \\ I_2 \\ I_3 \\ I_{12} \\ I_{23} \\ I_{31}
\end{bmatrix}
=
\begin{bmatrix}
0 \\ 0 \\ 0 \\ 0 \\ E_1 - E_2 \\ E_2 - E_3 \\ E_3 - E_1 \\ E_1 \\ E_2 \\ E_3
\end{bmatrix}
\tag{9}
$$

To solve the set of Equations (9) and find B = 10 currents, the program MATLAB was used. The following is the MATLAB script for the solution [11]:

```
E=230;E1=E;E2=E*exp(-j*2/3*pi);E3=E*exp(j*2/3*pi); ET=[E1 E2 E3]';
ZL1=0.2+j*0.1;ZL2=0.2+j*0.1;ZL3=0.2+j*0.1;ZN=0.3+j*0.1
Z1=7+j*4;Z2=5-j*5;Z3=5-j*5;
R1=10;R2=15;R3=5;XL1=8;XL3=10;Xc=5;XM=3;Z12=R1+j*XL1;Z23=R2-j*Xc;Z31=R3+j*XL3;
Matrix=[1 0 0 0 -1 0 0 -1 0 1
    0 1 0 0 0 -1 0 1 -1 0
    0 0 1 0 0 0 -1 0 1 -1
    0 0 0 -1 1 1 1 0 0 0
    ZL1 -ZL2 0 0 0 0 0 Z12 0 -j*XM
    0 ZL2 -ZL3 0 0 0 0 0 Z23 0
    -ZL1 0 ZL3 0 0 0 0 -j*XM 0 Z31
    ZL1 0 0 ZN Z1 0 0 0 0 0
    0 ZL2 0 ZN 0 Z2 0 0 0 0
    0 0 ZL3 ZN 0 0 Z3 0 0 0]
Right_Side=[0 0 0 0 E1-E2 E2-E3 E3-E1 E1 E2 E3]'
I=inv(Matrix)*Right_Side;
IL1=I(1); IL2=I(2); IL3=I(3);IN=I(4); IL=[IL1 IL2 IL3]'
I1=I(5);I2=I(6);I3=I(7);            Iy=[I1 I2 I3]'
I12=I(8); I23=I(9); I31=I(10);      ID=[I12 I23 I31]'
subplot(1,3,1),compass(IL),hold on,compass(IN,'b'),title('Currents I_L and I_N')
subplot(1,3,2),compass(Iy),hold on,compass(IN,'b'),title('Currents I_Y and I_N')
subplot(1,3,3),compass(ID),title('Currents I_D')
```

The results of a MATLAB script are shown below, in Table 1 and Figure 4.

Table 1: Values of line, star and delta currents.

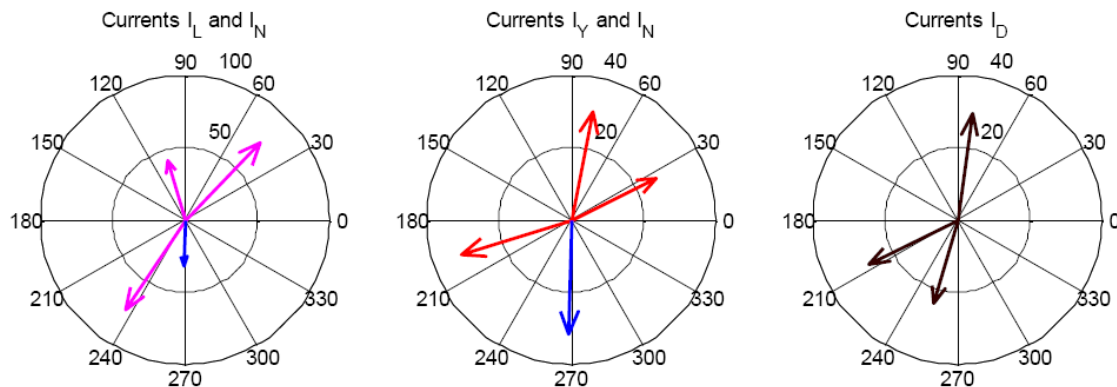| Currents of line I_L	Currents of star I_Y	Currents of delta I_Δ

I$_L$ =
  52.3218 +52.8851i
 -41.3931 -62.1979i
 -11.6005 +41.0082i
 	
I_N = -0.6718 -31.6955i
	I$_Y$ =
	  23.6713 +11.3950i
	 -30.3931 - 9.6992i
	  6.0500 +29.9997i
		I$_\Delta$ =
		  4.3626 +29.3305i
		 -6.6374 -23.1682i
		 -24.2880 -12.1597i

Figure 4: Phasor diagram for line currents I_L, star currents I_Y and delta currents I_Δ.

NONLINEAR CIRCUIT WITH TUNNEL DIODE

A nonlinear circuit consists of at least one nonlinear element, not including the independent voltage and current sources. This is in contrast with linear circuits, which consist of linear elements only Nonlinear circuits may possess multiple solutions or may not possess any solution at all. A circuit element is called nonlinear, if its constitutive relationship between the voltage and the current is a nonlinear function or a nonlinear relation.

Analysis of nonlinear circuits is more difficult than the analysis of linear circuits. Kirchhoff's Voltage Law and Kirchhoff's Current Law express linear relationships between a c ircuit's voltages or currents [12]. The nonlinear relations between circuit variables stem from the element relations. As a result, the equations describing the behaviour of a nonlinear circuit are nonlinear.

Consider the nonlinear circuit shown in Figure 5a. The circuit consists of a DC voltage source E, resistances R_1, R_2, R_3 and a tunnel diode, with a nonlinear characteristic (Figure 5b). The aim is to determine the diode current I_D and voltage V_D.

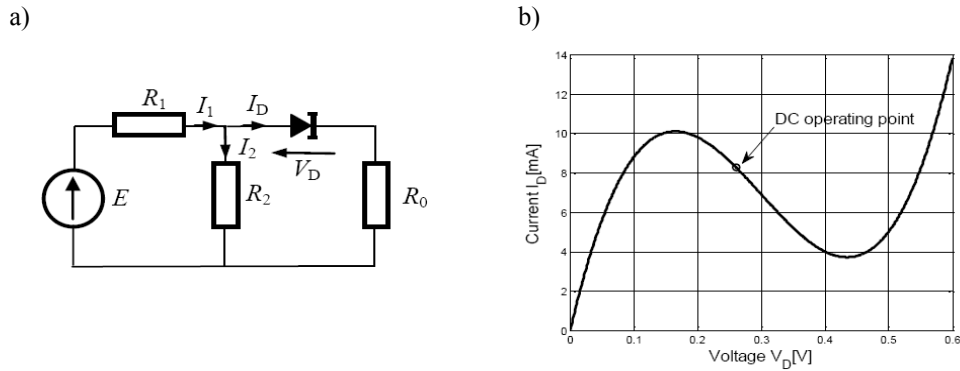a)   b)

Figure 5: a) Nonlinear circuit with tunnel diode; b) The approximated *i-v* characteristic of tunnel diode.

Applying KCL for node and writing KVL around the loop gives the following set of equations:

$$I_1 - I_2 - I_D = 0$$
$$E - R_1 I_1 - R_2 I_2 = 0 \qquad (10)$$
$$R_2 I_2 - V_D - R_2 I_D = 0$$

The diode current and voltage are related by the diode equation.

$$I_D = c_1 V_D^3 + c_2 V_D^2 + c_3 V_D \qquad (11)$$

The program MATLAB was used to solve the set of Equations (10) with the characteristic (11) to find the diode current I_D and the diode voltage V_D. The following MATLAB script was used to produce the solution:

```
function Eq=dioda_t(x)
global R1 R2 R3 C1 C2 C3 E1
%I1=x(1),I2=x(2),ID=x(3),VD=x(4)
Eq=[x(1)-x(2)-x(3)
    E1-R1*x(1)*10^-3-R2*x(2)*10^-3
    x(4)-R2*x(2)*10^-3+R3*x(3)*10^-3
    x(3)-C1*x(4)^3-C2*x(4)^2-C3*x(4)];

% Nonlinear circuit
global R1 R2 R3 C1 C2 C3 E1
R1=300;R2=150;R3=50;C1=650; C2=-585; C3=140;E1=4.5;
% charakteristic v-i of tunnel diode %ID=C1*VD^3+C2*VD^2+C3*VD
C=[C1 C2 C3 0];VD=0:0.001:0.6;ID=polyval(C, VD);
plot(VD,ID,'LineWidth',2.5,'color','k'),grid on, hold on,axis([0 0.6 0 14])
xlabel('Voltage V_D[V]'),ylabel('Current I_D[mA]')
%calculation of the nonlinear circuit
x0=[1 1 8 0.3]; x = fsolve(@dioda_t,x0);sol=x'
plot(x(4),x(3),'ok','LineWidth',2)
result=sprintf('I1=%0.3f mA, I2=%0.3f mA, ID=%0.3f mA, VD=%0.3f V',x)
```

The results of the calculation are presented below:

$I_1 = 12.755$ mA, $I_2 = 4.491$ mA, $I_D = 8.264$ mA, $V_D = 0.260$ V

CONCLUSIONS

In this article, the authors show how the Maple mathematic software program and MATLAB can be used effectively in the analysis of electrical circuits. Program Maple was used to formulate structural matrices of the loop system of electric circuit equations. Without performing many time-consuming computations, students are able to obtain fast results. The MATLAB code demonstrated in the article is relatively simple. It is easy to draw and animate the solution too. In the analysis of nonlinear circuits, students use the numerical solution. This helps students to understand that some problems can only be solved by numerical methods. Students learn how complex practical problems can be, and learn to appreciate the need for computer solving. The symbolic, numerical and graphical features of mathematical programs allow students to carry out mathematical analyses of electrical circuits and to explore easily their behaviour.

REFERENCES

1.   Chen, W.K., *The Electrical Engineering Handbook*. In: Dorf, R.C. (Ed), Boca Raton, FL: CRC Press (2000).

2. Irwin, J.D. and Nelms, R.M., *Basic Engineering Circuit Analysis*. Hoboken, NJ: Wiley (2008).
3. Chua, L., Desoer, C. and Kuh, E., *Linear and Nonlinear Circuits*. USA: McGraw-Hill Book Company (1987).
4. Dąbrowski, A.M., The investigation of dynamic states in electric circuits with Maple. *Proc 6th Baltic Region Seminar on Engng. Educ.*, Wismar, Germany, 79-82 (2002).
5. Dąbrowski, A.M., Mathematical programmes in teaching of circuit theory. *Proc. XVI BSE'2002*, Gliwice, Poland, 32-37 (2002).
6. Dąbrowski, A.M., Computer programmes in teaching of electrical engineering. *Proc. XVII BSE2003*, Istebna-Zaolzie, Poland, 128-133 (2003).
7. Mitkowski, S.A., Dąbrowski, A.M., Porębska, A. and Kurgan, E., Electrical engineering education in the field of electric circuits theory at AGH University of Science and Technology in Kraków, *Proc. 1st World Conf. on Technol. and Engng. Educ.*, Krakow, Poland, 47-53 (2010).
8. Swamy, M.N.S. and Thulasiraman, K., *Graphs, Networks, and Algorithms*. New York: Wiley Interscience (1981).
9. Monagan, M., Geddes, K. and Heal, K., Maple *Introductory Programming Guide*. Waterloo, Canada: Maple Inc. (2009).
10. Char, B.W., Maple User Manual. Waterloo, Canada: Maple Inc., (2009).
11. The MathWorks, Inc. *MATLAB® Mathematics* (2011).
12. Attia, J.O., *Electronics And Circuit Analysis Using Matlab*. Boca Raton, FL: CRC Press (2004).

BIOGRAPHY

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Stanisław A. Mitkowski was born in Kraków, Poland. In 1962, he commenced his studies in the Faculty of Electrical Engineering at the AGH University of Science and Technology (AGH-UST), Kraków, graduating Master of Engineering (Electrical) in 1968. He obtained a PhD and DSc (Doctor Habil.) from the same University in 1974 and 1989, respectively. He has been with the AGH-UST since 1968, initially as an assistant and, since 1992, as a Professor. He has held a number of important positions, including that of the Vice-Rector (Science) between 1990 and 1996, and he has been Head of the Department of Electrical and Power Engineering since 1993. He is an Honorary Professor of the National Mining Academy of Ukraine, Dnepropetrovsk, Ukraine. Prof. Mitkowski also held the position of Vice-Rector for Development at the HEPS in Tarnów (1998-2001), and is Director of its Polytechnic Institute. His research interests include circuit theory, in particular non-linear electric circuits and engineering education. He has published extensively on these subjects in national and international journals and conference proceedings. Prof. Mitkowski was Vice-President of the National Council of the Polish Society of Theoretical and Applied Electrotechnics, between 1993 and 2008, and is a member of the Branch of Theory of Electrotechnics of the Committee for Electrotechnics within the Polish Academy of Sciences, a member of the IEEE, a member of the Society of Polish Electrical Engineers, and a member of several other national and international organisations. Between 2002 and 2007, Prof. Mitkowski was a member of the State Accreditation Committee (SAC) in Poland. He was a member of the then UNESCO International Centre for Engineering Education (UICEE), and received the UICEE Silver and Gold Badge of Honour for his achievements in engineering education in 1998 and 2007, respectively. Presently, he is an active member of the World Institute for Engineering and Technology Education (WIETE). Prof. Mitkowski is also a passionate stamp-collector and Chairman of the Kraków Branch of the Society of Polish Philatelists.

Anna Porębska graduated Master of Science in Electronics and Master of Science in Computer Science from the AGH University of Science and Technology (AGH-UST) Krakow, Poland, in 1985 and 1988, respectively. She obtained her PhD in Electrical Engineering from the same University in 1996. Currently, she works in the Department of Electrical and Power Engineering at the AGH-UST. Her professional interests include circuit theory, parallel processing, cellular automata, cellular neural networks and engineering education. She is a member of the Polish Society of Theoretical an Applied Electrical Engineering.