INTRODUCTION

Mathematical problems have to be solved in order for a person to go through a maturing process and a means of self-maturation to ensure the existence and basic skills that should be owned by that person [1]. In some cases, students struggle to resolve the problems in their lives; this not only happened in the past, but also with situations that they encounter in the present, and may encounter in the future. However, one cannot predict the types of problems that will be faced. Some researchers have conducted studies that emphasis the view that the success of students’ problem-solving is an important factor in understanding the beliefs of mathematics; students’ beliefs toward solving the problem also depend on the motivation of the belief in mathematics [2-4].

Related to belief in mathematics, Schoenfeld defined ...an individual’s understanding and feelings that shape the ways that the individual conceptualises and engages in mathematical behavior [4]. Beliefs and perceptions are interchangeable in the context of the nature of mathematics. Mathematical beliefs are value judgments towards mathematics that a person has gained from past experience [5]. Mathematics belief is an individual understanding of the world of mathematics, while identifying shared mathematical tasks. It is not only the field of cognitive, but also affective fields, such as attitudes and beliefs that affect the stages of solving the problem [1][6][7].

Jonassen [1] and Lerch [8] stated that student success in solving a problem is directly proportional to their attitudes and beliefs towards solving the problem. In his study, Lerch found that the decision made by four students who became the subjects of research during the search for solutions to problems depends on the material content knowledge and belief system of students to mathematical problems [8]. Therefore, the belief in mathematics can be defined as the sum of the value judgments and someone’s subjective approach developed through the experience in learning and mathematical problem-solving.

Some studies focused on students and teachers have explained that the beliefs about mathematics influence the mathematics success of students while studying at school [2][4][5][7-11]. However, studies that focused on the belief of prospective teachers have been limited in number [12-15]. Within this scope, the interaction between variables, such as problem-solving level, academic ability and gender is seen in a different dimension from previous studies. For example, Kloosterman and Stage, in their study that examined problem-solving in relation to students’ beliefs, reached the conclusion that the mathematical belief of an individual affects learning and problem-solving [2]. On the other hand,
studies from Giovanni and Sangcap that focused on students concluded that belief in mathematical problem-solving is a low-level relationship involving basic arithmetical skills and simple solution strategies [14]. In solving the application of mathematical problems, operational skills are only at a mechanical problem-solving stage and giving the correct answer to the problem is enough for success in mathematics.

Some studies of belief in problem-solving defined beliefs through questionnaires or interviews [7][16-18]. In this study, belief in problem-solving has been measured using interviews and a task-based problem-solving approach. Interviews were used to describe the subject’s belief in solving problems in a comprehensive manner, so that the mathematical behavior of the subject in problem-solving can be observed, and their belief in the problem-solving process can be revealed by supporting explanations. Muhtarom established that 62% of prospective teachers believed in mastering a two-dimensional figure and 40% believed in mastering a circle tangent. Therefore, the researchers are interested in studying belief in the problem-solving process of prospective teachers with high mathematics ability on material relating to 1) two-dimensional figures; and 2) tangents to a circle [19].

RESEARCH METHOD

This type of research is qualitative-explorative and is conducted on the beliefs of prospective teachers with high mathematics ability in the problem-solving process. Subjects in this study were prospective teachers of Universitas PGRI Semarang with high mathematics ability. The process of selecting subjects was started by giving standardised tests of mathematical ability to the prospective teachers. The test results were processed, then, subjects were selected by choosing ones who had high mathematical ability with the test score ≥ 85. Instruments in this study were divided into three parts: 1) the researchers themselves as the main instrument; 2) the problem-solving worksheet (PSW); and 3) guidelines for an interview. Interviews were used to obtain the subjects’ belief to problem-solving in a detailed manner, then, the subjects’ belief was proven during the problem-solving process. PSW in this study are presented in Table 1.

<table>
<thead>
<tr>
<th>Matter</th>
<th>Problem</th>
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<tbody>
<tr>
<td>Two-dimensional</td>
<td>The width of square ABCD is 25 cm. Point E, F and G each of which is the midpt of AB, AD and CD. Suppose the intersection between BD and FG is a point of H. Find the width of BHFE.</td>
</tr>
<tr>
<td>figure</td>
<td>Two gears have each a radius of 90 cm and 30 cm. Both gears are known to intersect and are closely surrounded by a chain. Determine the length of the chain?</td>
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<tr>
<td>Tangent circles</td>
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</table>

The data collection process was started by interviewing the subjects relating to the belief in the process of solving the PSW. Results of transcription and observations were used to express the prospective teachers’ beliefs in the problem-solving process. These data expressed the subjects’ beliefs in what they believed. Next, the subjects were asked to practice solving mathematical problems. For problem-solving practice, the subjects were given PSW, asked to read it carefully and, then, make observations during the data collection process. Testing the credibility of the data obtained was done in three ways; namely, making observations continuously, consistently and unyieldingly (increase persistence), asking the subjects to reread or recheck the data that the researchers had obtained based on the data submitted by the same subject (member check) and time triangulation (data obtained at a variety of times).

Triangulation is a technique of data validity checking that utilises something else beyond the data for the purposes of checking or as a comparison against the data. The problem solved in the process of triangulation was a problem that was similar or equivalent to a problem that had been given previously. Then, the collected data were analysed by the following steps: 1) reduction of data, which is the process of election, simplification focusing, abstraction and transformation of raw data in the field. If there are invalid data, data which could be used as complementary data or sideline findings were collected separately; 2) presentation of data, that is, classifying and identifying data, is done, so it is well organised and categorised; and 3) conclusions based on the results of data presentation was reached. After the data were presented as such, the next step was to infer or interpret the meaning of the exposure data [20][21]. The data analysis was performed on any data obtained from each method of data collection (task analysis, interviews). At the end of the analysis, the problem-solving process of each subject and their beliefs was described in detail.

RESULTS AND DISCUSSION

The subjects in the study were given a code LW and researchers were coded TR. The code number indicates the sequence of stages of the interview (e.g. LW_10: this means that subject LW was tenth in the interview sequence).

Belief in the Problem-Solving Process on Two-Dimensional Figures

Subject LW believes that the problem had a high difficulty level, because she did not have an early picture of problem’s resolution and had to make connections from known information on issues, such as the midpoints of AB, CD, AD, finding the intersection between BD and FG. The subject believes that the concept needed to solve the problem was the Pythagorean theorem, the width of the two-dimensional figure, area and width of the triangle. It was based on the subject’s belief that the concept of Pythagoras’s theorem should be used to find the length of the line in the right-angled
Belief in the Problem-Solving Process on Circle Tangents

One subject believes that the problem has a high difficulty level because she must draw them in advance to make it easier to do it and only knows the radius only. So, it needs some other way to get it done. The subject believes that the concept needed to resolve the problem is half the circumference of the circle and the tangent to the circle equation. The concept of tangent lines should be used to search the length of the tangent to the circle (the length of the chain that touches a wheel), while the concept of the circle’s circumference should be used to find the circumference of the circle (length of chain attached to the wheels).

TR_02 After reading the problem and understanding it, what beliefs do you have related to this problem? Is its difficulty level easy, medium or difficult? Give reasons.

LW_02 In my opinion, this is difficult, because one had to draw it first to make it easier to solve and the only known here is the radius. So, it needs some effort to get it done.

TR_15 How do you believe that the two concepts will be used to solve the problem?

LW_15 For the concept of circles, one is here to find the circumference, and one uses the concept of the circle.

TR_16 Then?

LW_16 The concept of tangents, because the wheels touch each other so we use the concept of a tangent.
The subject believes that she can solve the problem with one way of settlement, which is the circumference of a circle with external tangents. Furthermore, the subject believes the stages of problem-solving begin with finding the circumference then summing the length. The subject believes that the concept, relationships between concepts, steps to resolve, and believes the truth of the answers that would be obtained from the settlement would be based on the truth of the concepts used in problem-solving.

LW_31 Stages can be described ... stages of completion that you believe can solve this problem. In stage one, one looks for the circle tangent, then, one finds the circumference. Then, for the length of the chain, the tangent plus circumference of the circle.

TR_32 First, seek the tangent, then, look for the circle circumference, and the length of the chain is the number.

LW_32 The sum of the tangent and the circumference of a circle.

TR_33 Do you believe the steps of such settlement will get the right answer?

LW_33 Sure. If I work in accordance with the mathematical concept from the beginning, the final result will be correct.

According to Subject LW’s belief in the problem-solving process on the two-dimensional figure as explained above, the subject believes one can find a solution based on the important information contained in the matter. The next step for the subject is the problem-solving process reflecting the implications of his belief (see Table 2).

Table 2: Problem-solving process of the subject on tangent circles.

<table>
<thead>
<tr>
<th>Transcript of subject speech</th>
<th>Written work</th>
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<tr>
<td>(Making sketch) this (large circle) of radius 90 cm and this (small circle) of radius 30 cm. Both gears are known to intersect and are closely surrounded by a chain. The tangent line AB is parallel to CE; to seek CE one uses the Pythagorean theorem. The length of DC is the sum of the two radii (CD = 30 + 90 = 120). CE = root of CD squared minus DE squared. Because, this (BC) is parallel to this (AE) = 30 cm, so DE = 90 cm - 30 cm = 60 cm. Then, CE is equal to the root of 120^2 minus 60^2, CE is equal to the root of 14,400 minus 3,600. CE is equal to the root of 10,800. The roots of 10,800 can be broken into three times 3,600. The roots of 3,600 are 60. So the outcome for the CE is 60√3 cm. Because AB = CE = 60√3 cm.</td>
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</table>

For the large circle, one looks for the length of the chain using a formula of the semicircular circumference. Then, the formula K = π r. π = 3.14 and r = 90 cm; semicircular circumference = 282.6 cm. For the small circle, r = 30 cm. Then, the formula K = π r. π = 3.14 is applied, and r = 30 cm. A semi-circular circumference = 94.2 cm. 

For the length of the chain one takes tangents AB, plus half of the circumference of the small circle, plus the length of tangent FG, plus circumference of half of the large circle. AB= 60√3 cm, plus circumference of the small semicircle = 94.2 cm, plus FG = 60√3 cm, plus the large semi-circular circumference of 282.6 cm. The length of the chain = 60√3 + 94.2 + 60√3 + 282.6 = 584.4 cm. |

Subject LW believes the correctness of the answers that would be obtained based on the concept, concepts relationship and phases of steps that should be used. The subject has been able to represent the problem correctly, that is any tangent to the circle is definitely perpendicular to the radius of the circle. But, in the process of solving mathematical problems on the material tangent circles, LW made a mistake in understanding the concept of half of the circumference of a circle.

The subject understands that to find the length of the chain using a semi-circular circumference, without considering the angle CDE, which cannot possibly be 90°. In fact, the subject believed that the CE^2 = CD^2 - DE^2; then, the value of angle CED is right-angled. This means that the value of the angle CDE may not be 90°, but the subject does not consider the connection between the information provided in the problem.

It is obvious that Table 3 shows that the belief of the subject is in accordance with the characteristics of the PSW problem, but it differs in their implications in the problem-solving process on the tangent to a circle. In this respect, motivation and metacognitive skills are associated with the belief in an attempt to solve the problem. This study shows that there is a complex relationship between the person’s belief system and the implications of solving the problem.

Understanding what is believed by someone cannot guarantee the correctness of the problem-solving process being used. Some believe the concept, the concept relationships and the stages of the problem-solving process, but the correctness of the answers obtained will also depend on metacognitive ability in evaluating any problem-solving process that has been done. This is seen in the problem-solving process on the tangent to the circle. This study has shown that one cannot directly make a causal relationship between belief in the problem-solving process (or vice versa).
Table 3: Comparison of belief in the problem-solving process.

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Belief in the problem-solving process of two-dimensional figures</th>
<th>Belief in the problem-solving process of tangent circles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belief in the level of difficulty</td>
<td>The subject believes that the given problem is categorised as difficult, because the subject has not had a preliminary picture of resolution. Thanks to the strong efforts and motivation, the subject finally is able to represent the problems correctly. It shows the motivation affects the subject’s effort in order to resolve the problem, in this case, is in understanding the problem.</td>
<td>The subject believes that the given problem is categorised as difficult because the problem information is very limited, with the only known being the radius of the circle. The subject strives to be able to represent the problem in the form of images correctly. This effort shows that the subject has been able to understand the problem properly.</td>
</tr>
<tr>
<td>Belief in the understanding of concepts related to the settlement of problems</td>
<td>The subject believes that the concept needed to solve the problem is the Pythagorean theorem, the area of two-dimensional figures and area of a triangle. The problem-solving process indicates that the concept is used in the search for the length-line in the right-angled triangle, the area of the triangle and determining the relationship of the square area with the area of the triangle. This indicates that there is correspondence between concepts that the subjects believe in and the problem-solving processes.</td>
<td>The concept required solving the problem of the tangent to the circle and the circumference of a circle. The problem-solving process indicates that the concept of a tangent is used to find the length of the chain that does not stick to the wheel, the concept of a circle circumference is used to find the length of the chain attached to the wheel. But, in the problem-solving process the subject made a mistake in finding the length of the chain attached to the wheel.</td>
</tr>
<tr>
<td>Belief in the planning of problem-solving</td>
<td>The subject believes that the stages of problem-solving using two methods are; namely, using the trapezoidal approach and the area of a triangle. The trapezoidal approach begins with finding the length of related lines, then, uses the trapezoidal formula to find the area of the shaded region. The approach of the triangle area begins with finding the relevant line length, the area of the triangle and, then, determining the area of the shaded region. This belief was shown by the subject in the process of solving the given problem.</td>
<td>The subject believes that to be able to solve the problem, one should start by looking for the circumference of the half-circle of the big circle and half circle for the small circle. Next is to search for the length of external tangents between the circles. This corresponds to the problem-solving process performed by the subject.</td>
</tr>
<tr>
<td>Belief in the correctness of the answers obtained</td>
<td>The subject believes the correctness of the answers obtained is based on the relevance of the concepts used and whether the answer makes sense in the context of the initial problem. This corresponds to the problem-solving process being done that is the subject uses all the information and concepts that are relevant and always conducts an examination of each stage of completion, so that the answers obtained are appropriate and in accordance with the given problem.</td>
<td>The subject believes the truth of the answers that would be obtained from the settlement is based on the truth of the concepts used in problem-solving. But, the problem-solving process shows that the work of the subject is less precise, because of mistakes made by the subject in the search for the length of the chain attached to the wheel.</td>
</tr>
</tbody>
</table>

The next issue that lecturers face is how to develop the beliefs of prospective teachers, so that they can solve mathematical problems. Educators including mathematics lecturers and education researchers are encouraged to explore the beliefs in the problem-solving process of prospective teachers with larger samples from various universities. Conducting similar studies on the belief of prospective teachers from different cultures, it can be realised whether there is a difference between the beliefs evaluated in accordance with given explanations. The results of this study can also be used as the initial foundation to conduct a more in-depth study about leveling prospective teachers’ beliefs in the problem-solving process.

CONCLUSIONS

The research results on beliefs in the problem-solving process on two-dimensional figures has strengthened previous studies [1][8]. One’s success in solving the problem is directly proportional to their belief in solving the problem. Lerch [8] stated that the decision made by students during the search for solutions to problems depends on the material content knowledge and the students’ belief related to mathematical problems.

One’s success in problem-solving is an important factor in understanding beliefs about mathematics; belief toward solving the problem also depends on the motivation of the belief in mathematics [2-4]. Prospective teachers must have a positive belief in the profession so as to understand the students’ perspective in problem-solving and can train students
so that they can solve mathematics problems [12][17][22][23]). Bal stated that prospective teachers who can successfully solve problems believe in the importance of mathematics and the need to understand, make the information connection of the problem [23].

REFERENCES


BIographies

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