

## Wolfram Mathematica as applied to the interactive visualisation of descriptive geometry problems

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**ABSTRACT:** The interactive visualisation of the solutions to descriptive geometry problems, as compared to the traditional approach, enriches the content of geometric education, introduces new opportunities in the educational process, and stimulates students' interest, both in the study of the discipline and in further research in descriptive geometry. To demonstrate this capability for the geometric and graphical training of students, several interactive projects were developed in the form of the source code and output data. Difficulties are discussed, as well as possibilities for further implementations of projects for geometric and graphical student training. It is noted that interactive projects cannot replace, but rather complement, the fundamental knowledge received by students during a traditional course at a technical university.

**Keywords:** Descriptive geometry, Mathematica, interactive projects

### INTRODUCTION

Research in engineering design is a way to explore, understand and use design knowledge to find solutions to problems [1]. Classical, differential, algebraic and descriptive geometry has many useful geometric applications [2]. Digital integration combining research areas and involving personnel, processes, users, and data creates conditions for scientific and technological breakthroughs. This can support scientific and economic development, particularly in the global mineral and raw materials sectors [3][4].

Relevant research was reviewed. Csima et al determined and visualised the isoptic surfaces for Platonic solids and for some semi-regular Archimedean polytopes [5]. An alternative definition of the isoptic surface of a shape was presented by Nagy et al [6]. Nieto and Ramos presented an approach to programming 3-D graphical representations using a computer algebra system [7]. Bobenrieth et al developed a sketch-based modeller that constructs a 3-D shape by combining a single descriptive sketch with minimal user intervention [8]. Kumar and Taber proposed a representation framework for fields that leverages integral properties to incorporate diverse field models into a single analysis pipeline [9].

The problem of improving the quality of the engineering training of students at technical universities is one of the main tasks of the higher education system [10-13]. Rao et al created a new efficient method for the optimisation of mechanical design problems [14]. Modern software allows students to expand their mastery in graphic discipline courses. The use of computer mathematics systems enriches the content of mathematical and geometry-graphics education and introduces new ideas into the organisation of the education [15-19]. The advent of new software tools has changed the way these courses can be taught [20]. Today's engineering students should be provided with these tools to help them learn and understand the material more effectively [20].

As a result of the analysis of existing works, suitable pedagogical examples were identified that also provided guidance for the development of suitable projects.

### SURVEY

The proposed projects are here divided into six sections: points, lines, planes; straight line and the second-order surface; plane and the second order surface; second order surfaces; rotating bodies and polyhedrons; polyhedrons. The Wolfram Mathematica system was applied to the development of these projects.

## Example Projects

The project, *1\_4\_Points\_Planes*, can be used by a descriptive geometry teacher during a lecture that includes such concepts as projection planes, octants, points and planes. The teacher/user can quickly change the points' co-ordinates, transparency of the projection planes and of the constructed plane. Sample source code for this project:

```
{p1,p2,p3,p4}={{0,0,0},{1,0,0},{0,1,0},{0,0,1}};
{A1,B1,C1,D1}={{2,4,5},{3,2,1},{-1,1,3},{4,5,3}}.
```

```
Pi1=InfinitePlane[{p1,p2,p3}];
Pi2=InfinitePlane[{p1,p2,p4}];
Pi3=InfinitePlane[{p1,p3,p4}];
```

```
{Graphics3D[{Text["A",{Ax,Ay,Az+1}],Text["B",{Bx,By,Bz+1}],Text["C",{Cx,Cy,Cz+1}],Text["D",{Dx,Dy,Dz+1}],
Text["I",{5,5,5}],Text["II",{-5,5,5}],Text["III",{-5,-5,5}],Text["IV",{5,-5,5}],Text["V",{5,5,-5}],Text["VI",{-5,5,-5}],
Text["VII",{-5,-5,-5}],Text["VIII",{5,-5,-5}],Text["Pi1",{9,9,0}],Text["Pi2",{1,0,9}],Text["Pi3",{0,9,9}]
PointSize[0.02].
```

```
Point[{{Ax,Ay,Az},{Bx,By,Bz},{Cx,Cy,Cz},{Dx,Dy,Dz}},VertexColors→{Red,Green,Blue,Black}].
Opacity[p2],Brown,ABC=InfinitePlane[{{Ax,Ay,Az},{Bx,By,Bz},{Cx,Cy,Cz}}].
PlotRange→10,Axes→True,AxesLabel→{x,y,z}],{Ax,-10,10},{Ay,-10,10},{Az,-10,10},{Bx,-10,10},{By,-10,10},
{Bz,-10,10},{Cx,-10,10},{Cy,-10,10},{Cz,-10,10},{Dx,-10,10},{Dy,-10,10},{Dz,-10,10},{p1,0,1},{p2,0,1}]
```

The user, with the help of controls, can set any co-ordinates (including negative) of the four points (or only one of them). It is possible to manipulate the rotation, magnification, reduction of this interactive space, the transparency of the projection planes, and the plane drawn through points A, B and C. If necessary, in the first lesson, the user can, for instance, set the co-ordinates of points B, C and D as zero and explain to the students the location of only point A in different octants by specifying different values and co-ordinates for this point (Figure 1).

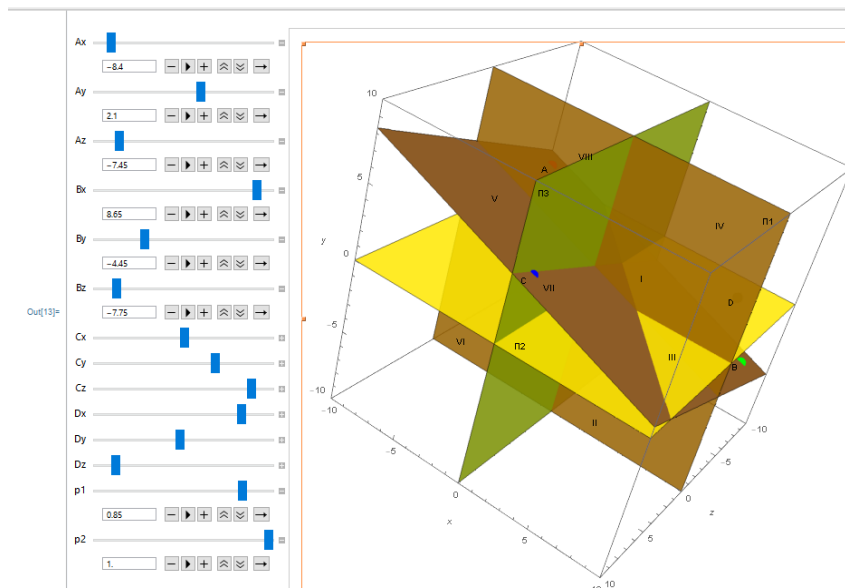


Figure 1: Project *1\_4\_Points\_Planes* shows a plane whose user-defined transparency is 1 and is drawn through plane points A, B and C; the user-defined transparency of the projection planes is 0.85.

The two-plane intersection line (*1\_2\_Planes*) is visualised in two ways: through graphic elements and through surface equations, using contour plots. There are versions of developed projects both with the designation of projection planes and octants, and without this designation; as well, there is transparency control of the projection planes.

The transparency of the projection planes can be changed with the help of  $p$  control. When the intersecting planes are made invisible, the user can use the interactive plot from the first lessons, explaining to students what projection planes and octants are.

The first project output for the construction of the two planes intersection line is expressed through graphic elements and visually represents to the user the construction of planes, and, accordingly, their intersection lines, with the equations of surfaces  $ax + bx + cx = d$ . Each plane is set using 4 parameters: coefficients  $a$ ,  $b$ ,  $c$  and  $d$ . The coefficients  $a$ ,  $b$ ,  $c$  and  $d$  can be changed with the help of controls. Plotting is done using surface equations, giving the user the opportunity to change the coefficients of the equation, which results in the rearrangement of the intersecting planes, and, accordingly, the line of their intersection. To make it easily visible, the line which crosses the plane is bright green (Figure 2).

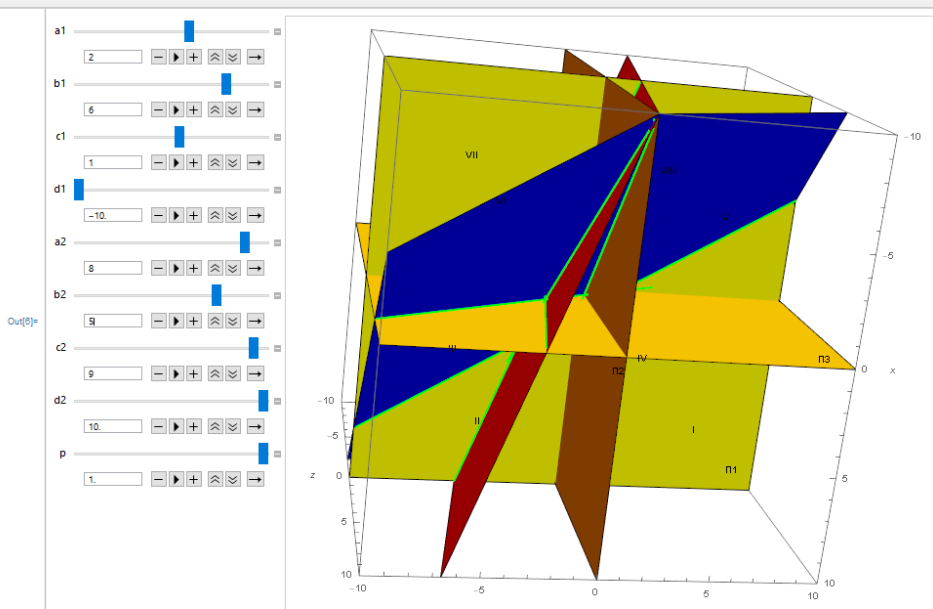


Figure 2: Project 1\_2\_Planes.

The main difficulty in working with surface equations was the need to ensure that when dynamically changing the depicted planes, the intersection lines of the planes also dynamically change. When working with equations, plane intersection lines are defined through areas, but when there are many lines it takes longer to recalculate the image and the diagram freezes up. Additionally, when working with graphic elements the dynamic rearrangement of the intersection plane line is much faster, but in the absence of co-ordinate axes and co-ordinate limits, the graph may be difficult to work with (due to a sharp change in scale).

Shown in Figure 2 is an option when the projection planes are demonstrated as brightly as possible. But at the same time, the intersecting planes themselves are visible. The line of the plane intersection and traces of the planes themselves are clearly shown.

Similar to the construction of plane intersection lines, it is possible to demonstrate, through interactive dynamic graphs of second-order surfaces, elements of their intersection with straight lines and planes (Figure 3).

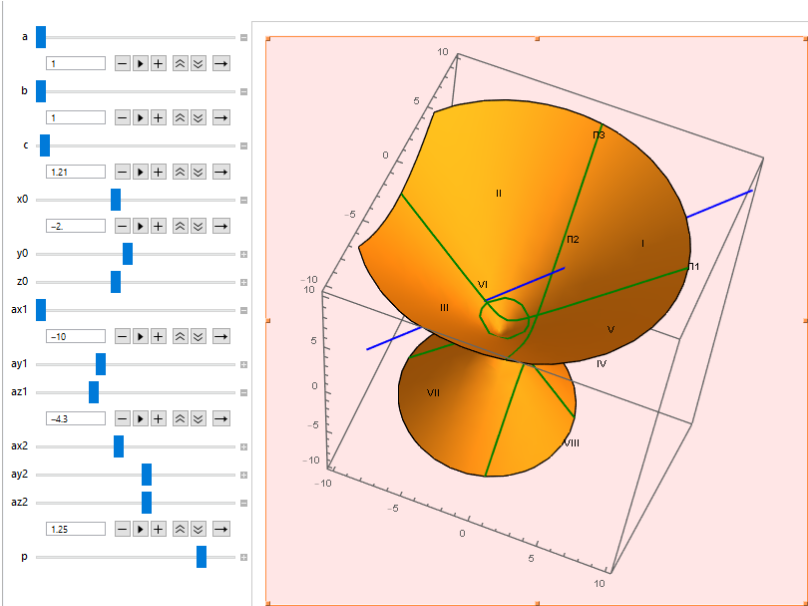


Figure 3: Project 2\_1\_Cone Surface

Also, clearly demonstrable is the relative position of second-order surfaces and polyhedrons (Figures 4-7). In the projects, the surface is defined through the function *CounterPlot3D* when using a Cartesian co-ordinate system.

For the interaction between a plane and surface, the plane is defined with the *infinite line* function, within which the plane parameters are specified. This method is almost completely similar to setting a plane using the function *CounterPlot3D*. In graphs of this type, only the intersection line of the surface with the plane (in red) is coloured. This is

done through the function *MeshFunction* inside the function *CounterPlot3D*. It would be beneficial to have the ability to colour differently this line and the line of the surface intersection with the projection planes.

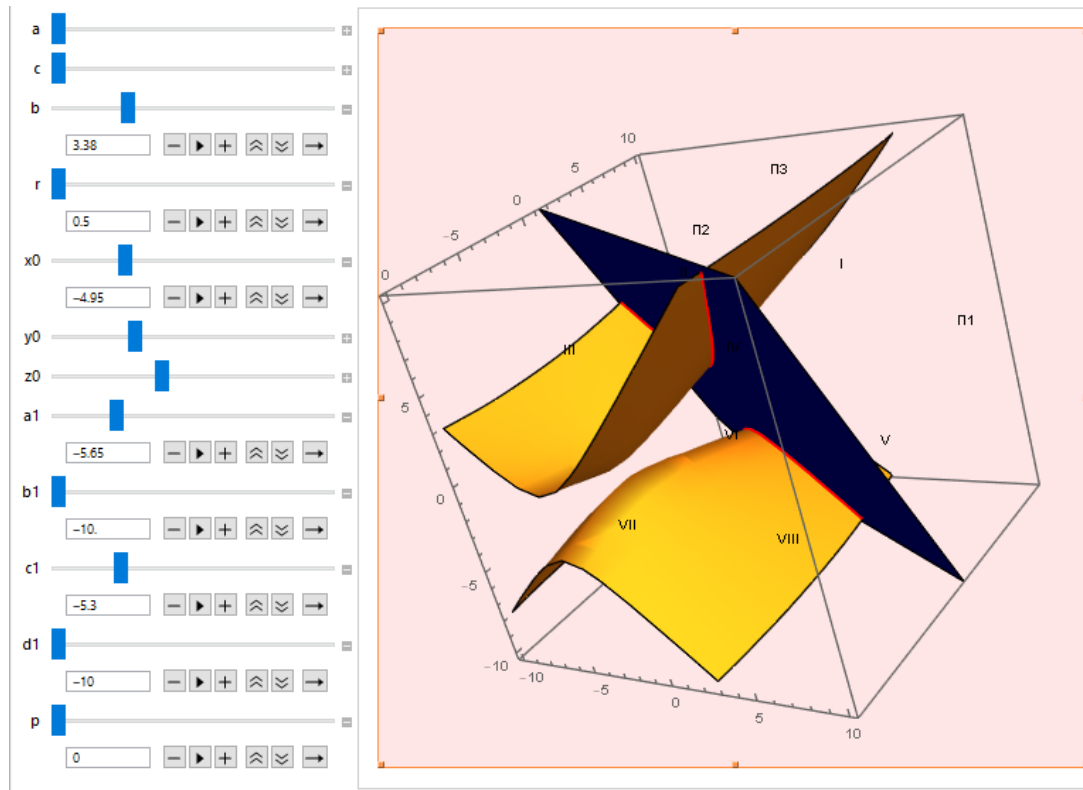


Figure 4: Project 3\_8\_Two-Surface Hyperboloid.

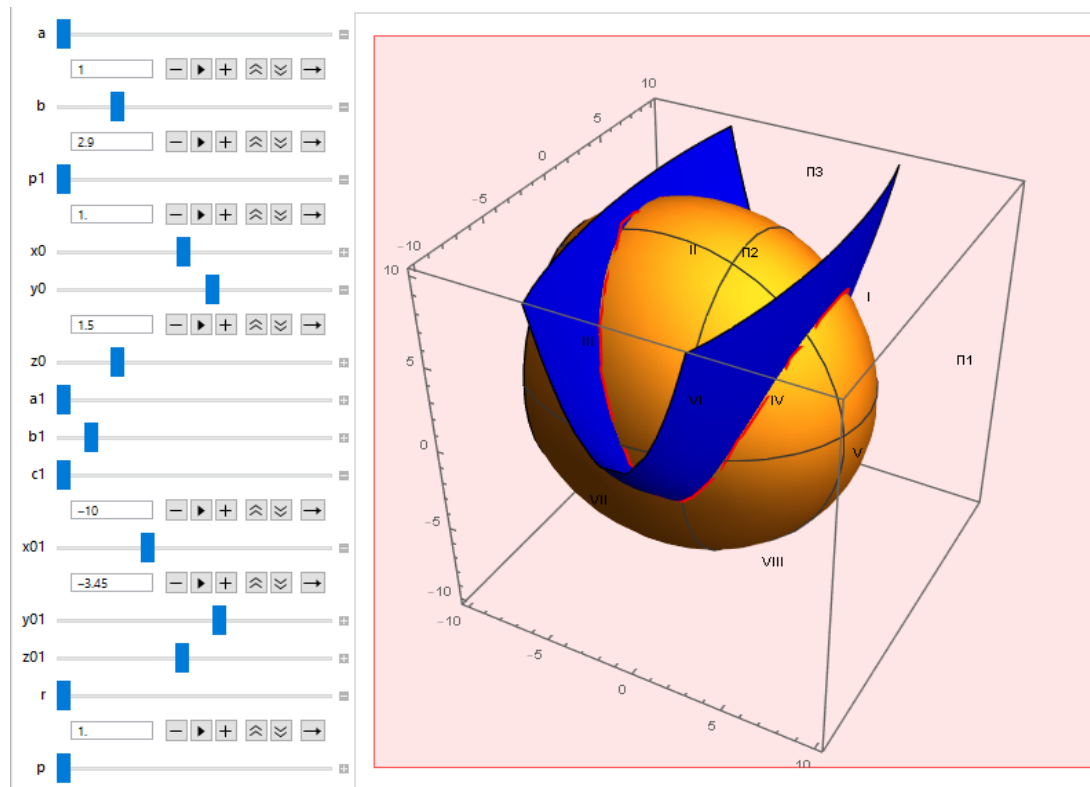


Figure 5: Project 4\_3\_Ellipsoid and Elliptical Paraboloid.

Then, the following elements of graphics were selected: ball, cone and cylinder to be used as the rotating bodies. Cube, tetrahedron, dodecahedron, octahedron and icosahedron (regular polyhedron) have been chosen as interactive polyhedrons. Four parameters (centre co-ordinates and edge length) are adequate. Graphic elements were the most convenient option. It was not possible to clearly colour the intersection of objects, but the added controls allow the manipulation of the transparency of each object (controls p1 and p2). In this case, with colour adjustment the intersection area will be quite noticeable.

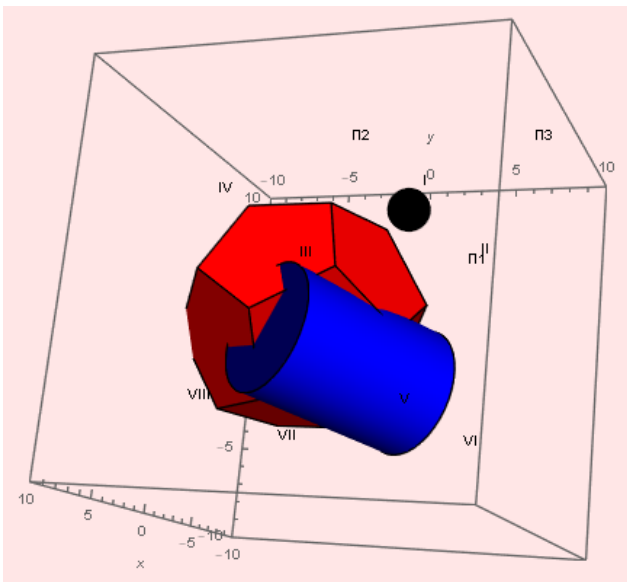


Figure 6: Project 5\_4\_Cylinder and Dodecahedron.

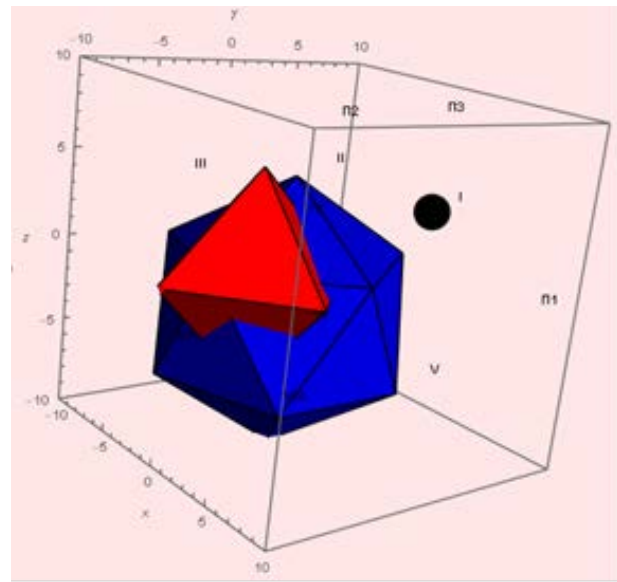


Figure 7: Project 6\_3\_Octahedron and Icosahedron.

## CONCLUSIONS

The authors created and, in this article, discuss the source codes and visual output data for interactive, static and dynamic projects, in the form of interactive graphs. They provide visual solutions to basic problems typical of a descriptive geometry course. These problems are grouped into six sections.

Teachers of descriptive geometry and graphics can use projects to create extra support for the educational process to be included in electronic textbooks, individual tasks and in automated systems for checking individual and control tasks. As well, they can be used as demonstration support for classes, i.e. interactive projects, presentations to exhibit various geometric objects during lectures, laboratory work and practical classes. Hence, this will facilitate the proper absorption of the material by students.

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## BIOGRAPHIES



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