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A competency-based calculus teaching module for engineering students

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ABSTRACT: The purpose of this study was to develop a competency-based calculus teaching module for engineering freshmen at a university of technology. This practical teaching module emphasises the connections between calculus and the technical information in engineering courses. The objectives of the teaching module were to provide engineering students with the ability to utilise the concepts and skills of calculus to conceptualise, formulate and solve technology problems, and to identify underlying patterns and structures. The duration of this programme was three years. Two classes of the Department of Electrical Engineering participated in this study, one class (Group A) learned with traditional teaching method and material, the other class (Group B) learned with competency-based calculus teaching modules. After the calculus course, the author documented the differences in these students' performance in the *Signals and Systems* courses. The new teaching module has proved to be effective.

INTRODUCTION

Engineering can be described as the use of mathematical, physical and technological knowledge to find solutions to challenging problems [1]. There is no doubt that mathematical skills are fundamental and indispensable to the practice of engineering professionals. But, traditional instruction methods emphasise hands-on skills. The author found that students lacking a solid theoretical foundation are able to learn technical information easily. However, in Taiwan, it has been found that without a solid understanding of mathematical concepts and skills behind the technology phenomena, it is not practical for the students to memorise all the technical information, and to apply these concepts and skills to the courses in engineering technology.

The lack of higher-level skills in mathematics and physics degrades the outcomes of the instruction provided. First, students forget the technical information, because the amount of information retained by them declines substantially after ten minutes [2]. Second, students know how things work but they do not know why things work. Students learn what they care about and remember what they understand [3]. Third, without appropriate requirements of logical reasoning and theoretical derivations, students take a long time to complete understanding of new concepts. This situation affects the outcomes of the course. The skills students have learned in this course and those in other courses cannot be effectively incorporated into each other. The isolation affects the outcomes of the whole curriculum.

Engineering education programmes are moving from an *input* to an outcomes' paradigm. Success is now focused on how well students achieve desired learning outcomes, not simply whether they have completed required coursework. The ABET 2000 Engineering Criteria 3 Programme Outcomes and Assessment have provided engineering programmes with the impetus and opportunity to re-craft how they educate students [4].

The concept of integral is an important part of the calculus curriculum in many countries. Indeed, it is not possible to imagine modern scientific culture without integrals. Along with its relative, the derivative, the integral forms the core of a mathematical domain that is a language, a device, and a useful tool for other fields, such as physics and engineering. Moreover, the concept of the integral represents a philosophical idea for understanding the world: contemplation of the totality of the small parts of a whole enables conclusions regarding the whole in its entirety, as well as its internal structure and properties. The idea of integrals emerged from within physics, from the attempt to invent a mathematical tool that enables describing, analysing and explaining physical phenomena, such as motion, mass and work. However, studies have documented that most first semester calculus students do not emerge from the course with an understanding of this concept; nor do they appear to be developing the foundational reasoning abilities needed to understand and use the concept in applied settings [5][6]. Student difficulties with the concept of the integral have been attributed primarily to their impoverished view of functions [7] and rate of change [8]. However, little research articulating what is involved in knowing and learning this concept is available. The purpose of this article is to provide additional clarity about the understandings and reasoning abilities involved in learning and using the concept of the integral. It also reports the results of a study that investigated the effectiveness of curricular materials for first semester calculus students that were developed using this framework as a guide.

METHODOLOGY

This programme was of two years' duration. Eighty five students from the Department of Electrical Engineering at a university of technology in Taiwan participated. There were two classes of students in this study, one class (Group A, 42 students) learned with traditional teaching method and material, the other class (Group B, 43 students) learned with *competency-based* calculus teaching modules. The abilities of students in these two classes were found to be equal through a pre-calculus concept assessment. After the instruction about integrals, the author wrote a post-instruction assessment, and continued to document the performance differences between these students in the *Signals and Systems* course. Both qualitative and quantitative research techniques, such as field notes, document analysis, pre-test and post-test, were used to investigate responses of students. The mean score and a number of correct responses for each item were compiled.

Each module contains in-class and take-home activities designed to promote the development of students' conceptual connections and reasoning abilities relative to the central concept of the module. Carefully sequenced prompts and tasks (situated in contexts) were included to promote students' articulation of their thinking. Instruction during the first two weeks of the semester included a strong focus on the foundational reasoning and understandings described in Part A of the framework. Post-instruction assessment of these understandings suggested that most students emerged with improved reasoning abilities and understandings. Instruction leading up to the module included a balanced focus on concept development, acquisition of notational understanding, facts and procedures, and the development of students' mathematical practices and problem-solving behaviours. Students were expected to be regular participants in the classroom. Whole class discussion, group work and lectures were the primary modes of instruction.

The class met twice weekly for 1.5 hours each meeting between 2 February and 4 March 2013, for a total of 10 meetings. Students had ready access to a computer laboratory or had a computer at home on which to work on assignments. The last two meetings - those in which the Fundamental Theorem of Calculus was discussed - were videotaped and transcribed. A small-group session after the last meeting was also videotaped and transcribed. The teaching experiment was structured to focus on four phases of conceptual development. These were:

- Phase I: Analyse behaviour of functions' graphs; explain their behaviour; model situations using functions and derive information about situations from graphs (3 meetings).
- Phase II: Average rates of change; functions which give average rates of change over all intervals of a fixed length (2 meetings).
- Phase III: Accumulations of change: Riemann sums (2 meetings).
- Phase IV: Relationships among variable quantity, accumulation of change and rate of change of accumulation (2 meetings).

The first phase focused on orienting students to reconstitute their images of functions, so that it would be based on images of co-variation. The second phase focused on having students enrich their notion of average rate, so that they could express it as a difference quotient that reflected average rate of change over an increment of some quantity. The third phase focused on having students conceptualise Riemann sums as functions that describe an approximate accumulation of one quantity with respect to variations in another.

The unit was intended to culminate in the fourth phase by asking students to bring these separate developments together in the context of problems that highlighted the inverse relationship between accumulation and accrual, so that they would have an opportunity to construct the concept of integrals. It was hoped that students would construct the integral; the larger aim of the teaching experiment, however, was to highlight aspects of their conceptions and orientations that might facilitate or obstruct such a construction.

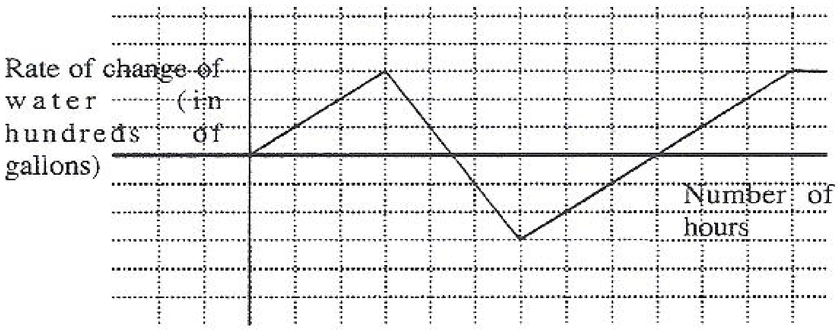
RESULTS AND DISCUSSION

Data selected from the post-instruction written assessment of students' understandings related to the integral and the *Signals and Systems* course are reported below. The presentation of results provides a statement of the item, the number of students who provided a correct response and the mean score on each part of each item.

Students' Performance in the Post-Instruction Written Assessment

Item 1: The Water Problem

Let *f* represent the rate at which the amount of water in Phoenix's water tank changed in (100's of gallons per hour) in a 12 hour period from 6 am to 6 pm last Saturday (Assume that the tank was empty at 6 am (t = 0)). Use the graph of *f*, given below, to answer the following:



	Number correct		Mean score (out of 3)	
	Group B	Group A	Group B	Group A
a. How much water was in the tank at noon?	39	26	2.8	1.7
b. What is the meaning of $g(x) = \int_0^x f(t)dt$	43	32	3.0	2.6
c. What is the value of $g(9)$?	40	29	2.8	1.9
d. During what intervals of time was the water level decreasing?	41	32	2.7	2.5
e. At what time was the tank the fullest?	35	22	2.3	1.4
f. Using the graph of *f* given above, construct a rough sketch of the graph of *g* and explain how the graphs are related.	35	22	2.4	1.6

The collection of responses on Item 1 suggests that the beginning calculus students in this study were proficient in applying co-variation reasoning with accumulation tasks. Over 80% of the students in Group B completing the course provided a completely correct response to parts d, e and f, suggesting proficiency in coordinating the accumulation of a function's input variable with the accumulation of instantaneous rate of change of the function from some fixed starting value to some specified value (MA3). Over 90% of these same students also provided a correct response to the prompts that assessed students' understanding of the notational aspects of the integral (parts b and c).

Item 2: The Circle Problem

Consider a circle that expands in size from $r = 0$ to $r = x$. Let *A* be a function that represents the accumulation of the rate of change of the circle as it increases in size from $r = 0$ to $r = x$.

	Number correct		Mean score (out of 3)	
	Group B	Group A	Group B	Group A
a. Define $A(x)$ as an accumulation function.	37	20	2.3	1.8
b. Construct a circle and illustrate what $\int_2^4 2\pi r dr$ represents.	41	24	2.8	2.1
c. Describe what $\frac{d}{dx}\int_a^x 2\pi r dr$ represents relative to the circle.	39	21	2.6	1.6
d. Construct the graph of *f* (the rate of change of the area of a circle), on the axes on the left and the graph of *A* (as defined above) on the graph on the right. Label your axes.	38	22	2.6	1.7
e. Explain how the two graphs are related.	35	18	2.2	1.5
f. Construct the graph of *A*. Estimate the area under the graph of *A* from $r = 1$ to $r = 5$ using eight approximating rectangles and right endpoints.	38	20	2.5	1.9
g. Given that *n* represents the number of subdivisions on the interval from $r = 1$ to $r = 5$, explain what is involved in letting the $\lim_{n\to\infty}$ for this interval.	39	23	2.6	2.0
h. What is the result of this evaluation?	38	24	2.7	2.1

Student responses on Item 2 also suggest that this collection of students in Group B possessed both a strong understanding of notational aspects of the integral (parts a, b) and proficiency in applying co-variational reasoning with accumulation tasks (parts f, g, and h). Responses on parts c, e and i indicate moderate proficiency in understanding the statement of the concept of integrals.

Item 3: The Distance Problem

A particle moves along a line so that its velocity at time *t* is $v(t) = t^2 - t - 6$ (measured in feet per second). Find the distance travelled during the time period from $t = 1$ to $t = 4$. Show Work!

Performance	Group B	Group A
Number correct	40	28
Mean score (out of 10)	7.2	5.8
Number of students who set up the integral correctly	42	26

Results for Item 3 reveal that most of the students in Group B recognised this question as an application of the integral. They were also successful in translating the situation to symbols. However, most students in Group A had difficulty recognising that they needed to sum the distance travelled in both the positive and negative directions.

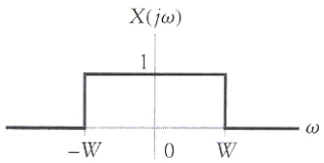
The collection of student responses on these items suggests that most of the students in Group B on completion of the course emerged with proficiency in using and understanding notational aspects of the concept of integral. These results also suggest that these students were able to apply co-variation reasoning with accumulation tasks. However, the students in Group A understanding of the statements and relationships of the integral tasks were weaker, with only a little over half of the students on completion of the course provided correct responses to the collection of questions assessing this ability.

Students' Performance in the Signals and Systems Course

Item 1: Inverse FT of a rectangular spectrum

Find the inverse FT of the rectangular spectrum depicted as follows and given by:

$$X(jw) = \begin{cases} 1, -W \prec w \prec W \\ 0, |w| \succ W \end{cases}$$



Performance	Group B	Group A
Number correct	40	28
Mean score (out of 10) of engineering knowledge	7.2	5.8
Mean score (out of 10) of continuous learning	7.9	6.1
Mean score (out of 10) of analysis and judgment	8.1	5.9

Item 2: ROC of a sum of exponentials

Consider the two signals:

$$x_1(t) = e^{-2t}u(t) + e^{-t}u(-t)$$

and

$$x_2(t) = e^{-t}u(t) + e^{-2t}u(-t)$$

Identify the ROC associated with the bilateral Laplace transform of each signal.

Performance	Group B	Group A
Number correct	38	26
Mean score (out of 10) of engineering knowledge	8.3	5.6
Mean score (out of 10) of continuous learning	8.1	5.7
Mean score (out of 10) of and analysis and judgment	8.4	5.9

Results for Items 1 and 2 reveal that the idea of an accumulation function has close connections with the main calculus ideas, function and integral. The idea of accumulation contributes to a coherent understanding of rate of change [7][8].

Two reasons in favour of using the accumulation approach in integral curricula are:

- The idea of accumulation allows for the combination of the concepts of definite and indefinite integrals in a natural way and to establish the connection between the (combined) concept of integrals and the concept of derivatives.
- The idea of accumulation is closely linked to the applications of the integrals.

The author believes that conceiving the integral as accumulation might help students answer conceptual questions, such as those posed in the questionnaire, correctly and with understanding. The question about existence might be more accessible to accumulation students than to common students, the value of the accumulating quantity is defined and may be calculated and summed up. Accumulation students are, thus, given opportunities to experience that the existence of the integral as accumulation depends only on the properties of the given function.

CONCLUSIONS

Over the years, the mathematics education community has tried with varying success to describe how students learn about the concept of integrals and point to better ways of teaching it. The fact remains that many students experience difficulties with the integral concept. The common approach to the integral enriches students' mathematical culture by affording them opportunities to use the integral in a formal way, mainly to calculate areas. However, the educational potential of the integral concept is much more significant. The realisation of this potential depends on the achievement of a deep comprehension of the concept. This begs the question of whether an approach to teaching the integral concept, which supports the development of such comprehension without neglecting technical skills, exists.

In this article, the author suggests that such an approach exists and brought forward evidence to support this suggestion. Significant comprehension of mathematical concepts is at the heart of mathematics learning. This proposition is particularly relevant for the integral. The idea of calculus, in general, and the idea of the integral, in particular, were born from human attempts to understand the world from applications.

In some way, the integral is the application. So, there is no way to understand integrals without understanding the strong connection between the mathematical concept and its applications. The heart of this connection is the idea of accumulation. Hence, the central aim of this research is to design a unit based on accumulation aimed at deep comprehension of the integral concept, and to investigate the resulting knowledge constructing processes. The first evidence gathered from a pilot implementation suggests that it will be possible to propose a suitable research-based didactical approach to teaching and learning the integral concept at the advanced high school level.

There are clear advantages for students in competency-based learning models. Because learning can be described and measured in ways that are comprehended by all parties, competency-based systems permit the learner to return to one or more competencies that have not been mastered in a learning process rather than facing the unwelcome prospect of repeating one or more traditional courses. Competency-based systems also provide students with a clear map and the navigational tools needed to move expeditiously toward their goals. In an ideal world, new competencies would logically and clearly build on other competencies. Competency-based systems have the potential to redistribute the power relationships between teachers and those taught [9]. Fortunately, some practical guidance to those institutions that wish to pursue competency-based models exist.

The data suggest that most of the first semester calculus students in this study completed the course with a strong understanding of notational aspects of integrals. They also demonstrated an ability to coordinate the accumulation of a function’s input variable with the accumulation of instantaneous rate of change, from some fixed starting value to some specified value, for various contextualised situations. The performance of this collection of students relative to the attributes of accumulation and the integral expressed in this framework were relatively good, especially, if this is compared with what has been reported of secondary teachers and graduate students [8].

By interpreting the ABET (a-k) Criterion 3 Programme Outcomes in terms of competencies, the author is committed to transforming the curriculum into the one built on competency-based learning. Such a curriculum would prepare students for the professional practice of engineering more effectively. Students would benefit from developing the competency that is necessary for success in the engineering workplace. Such competency would allow them to make connections across the entire curriculum and see their academic experience as an integrated whole rather than just a series of classroom requirements.

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