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# Research on engineering applications of Fresnel reflection functions

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ABSTRACT: Fresnel functions were obtained for the electromagnetic field at media interfaces. The precision Fresnel reflection function (FRF) was defined by averaging the parallel and perpendicularly polarised electromagnetic waves. Three engineering Fresnel reflection functions (EFRFs), i.e. the Schlick FRF, the simplified and the S-polarised EFRF were introduced. Through simulations, the three EFRFs were compared with the precision FRF. The Schlick FRF was the most accurate of the EFRFs and, hence, is the most useful. The results should improve understanding of FRFs and their application by university engineering students.

## INTRODUCTION

Electromagnetic wave polarisation is a basic concept, and essential properties of many materials will be affected depending upon the polarisation of an interacting electromagnetic wave [1][2].

The Fresnel reflection function (FRF) at the surface of a medium was determined using electromagnetic wave polarisation [3]. The FRFs are used in many fields, e.g. boundary theory and multilayer dielectric theory in optics and electromagnetics.

The FRF is the basis of the widely used bidirectional reflectance distribution function [4]. On the other hand, in engineering, it is necessary to use the engineering Fresnel reflection function (EFRF). The study of EFRFs is important both theoretically and practically.

## DIFFERENT FORMS OF THE FRESNEL REFLECTION FUNCTION

### Fresnel Equations

At a media interface, an incident electromagnetic wave is partially reflected and partially refracted (transmitted). The incident, reflected and refracted electromagnetic wave amplitudes spread out according to Fresnel's theory.

Electromagnetic waves are transverse, and only the electric field is visual; in fact, the light vector is exactly the electric field vector. For an electromagnetic wave in free space, the $\vec{H}$ (magnetic) and $\vec{E}$ (electric) fields have the same energy and since $\vec{H}$ can be expressed in terms of $\vec{E}$, only $\vec{E}$ will be considered.

To simplify the discussion without any loss of generality, the incident electromagnetic wave will be treated as two mutually perpendicular, linearly polarised electromagnetic waves, of the same intensity, i.e. $\vec{E}_s$ (S-polarisation), and $\vec{E}_p$ (P-polarisation).

The electromagnetic field boundary conditions can be determined by applying Maxwell's equations. This yields the composite wave obtained by superimposing the incident and reflected waves in the incident space in medium 1 and the refracted (transmitted) wave on the other side of the interface in medium 2. For non-metallic media, the total tangential components of the electromagnetic fields are continuous at the interface. From this can be derived the Fresnel formulae. For an electromagnetic field perpendicular to the incident plane, the Fresnel equations [5] are:

$$r_s = \frac{\cos\theta_i - n_r \cdot \cos\theta_t}{\cos\theta_i + n_r \cdot \cos\theta_t} = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)} \tag{1a}$$

$$t_s = \frac{2\cos\theta_i}{\cos\theta_i + n_r \cdot \cos\theta_t} = \frac{2\sin\theta_t \cos\theta_i}{\sin(\theta_i + \theta_t)} \tag{1b}$$

Where: $n_r = \sqrt{\varepsilon_{r2} / \varepsilon_{r1}} = n_2 / n_1$ [6],

is the relative refractive index of two dielectrics; $\theta_i$ , $\theta_t$ are the local angles of incident and of the transmitted electromagnetic waves. By Snell's law, the refractive and incident angles satisfy $\sin\theta_t = \sin\theta_i / n_r$ [7]; $r_s$ and $t_s$ are the reflection and transmission coefficients; s and p refer to S-polarisation and P-polarisation and are so used hereafter.

As for the electromagnetic field parallel to the incident plane, the Fresnel equations are:

$$r_p = \frac{n_r \times \cos\theta_i - \cos\theta_t}{n_r \times \cos\theta_i + \cos\theta_t} = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)} \tag{2a}$$

$$t_p = \frac{2\cos\theta_i}{n_r \times \cos\theta_i + \cos\theta_t} = \frac{2\sin\theta_t \cos\theta_i}{\sin(\theta_i + \theta_t)\cos(\theta_i + \theta_t)} \tag{2b}$$

From Formulae (1a) and (2a), by averaging the power reflectance with orthogonal polarisations obtained is:

$$F = \frac{\left(r_s^2 + r_p^2\right)}{2} \tag{3}$$

Where the function *F* is often called the precision FRF.

S-Polarisation Engineering Fresnel Reflection Function

Taking account of polarisation, the Fresnel functions [8] also can be expressed as follows:

$$F_s = \frac{a^2 + b^2 - 2a\cos\theta + \cos^2\theta}{a^2 + b^2 + 2a\cos\theta + \cos^2\theta} \tag{4a}$$

$$F_p = F_s \frac{a^2 + b^2 - 2a\sin\theta\tan\theta + \sin^2\theta\tan^2\theta}{a^2 + b^2 + 2a\sin\theta\tan\theta + \sin^2\theta\tan^2\theta} \tag{4b}$$

In addition:

$$\begin{cases} 2a^2 = \sqrt{\left(n^2 - k^2 - \sin\theta\right)^2 + 4n^2k^2} + \left(n^2 - k^2 - \sin\theta\right) \\ 2b^2 = \sqrt{\left(n^2 - k^2 - \sin\theta\right)^2 + 4n^2k^2} - \left(n^2 - k^2 - \sin\theta\right) \end{cases}.$$

For a metal, the refractive index generally is complex, i.e. $n + ik$ ; for non-metallic cases $k = 0$ and the refractive index is real [9]. As for the non-polarised case, the FRF takes the form:

$$F = \frac{\left(F_s + F_p\right)}{2} \tag{5}$$

This is called the precision FRF.

The Brewster [10][11] angle in Equation (5) is $F_p$ .

This plays a secondary role, and is ignored in approximate calculations where only the S-polarisation $F_s$ is considered, and is referred to as the S-polarisation EFRF.

Schlick Engineering Fresnel Reflection Function

The incident wavelength, incident angle and material optical properties are related to the reflection coefficient. The material optical properties, including refractive index $n(\lambda)$ and the attenuation coefficient $k(\lambda)$ are influenced by the wavelength.

Materials are divided into two categories: dielectrics and metal conductors. For dielectrics, in the visible light range, $k(\lambda)$ is 0; for conductors, $k(\lambda)$ is not 0.

Sometimes, it is necessary to examine the optical property of a material, but this may be hard to determine. However, when an electromagnetic wave has a normal incidence (i.e. $\theta = 0$), an estimate of the Fresnel reflection coefficient can be easily determined. Starting with $F(0,\lambda)$, Schlick [12] summarised the approximation solution $F(\theta,\lambda)$ as:

$$F(\theta,\lambda) = F(0,\lambda) + (1-\cos\theta)^5 [1-F(0,\lambda)] \quad (6)$$

This is known as the Schlick EFRF. In particular, $F(90^0,\lambda)=1$, is important for the case of grazing incidence.

Simplified Engineering Fresnel Reflection Function

Using the precision FRF for actual calculations is cumbersome. In engineering calculations, to reduce the amount of computation, the following approximation is usually used:

$$F_{\text{simple}} = (1 - A_0 \cos\theta_i)^2 \quad (7)$$

$F_{\text{simple}}$ is called the simplified EFRF, here: $A_0 = 1 - \frac{n_r - 1}{n_r + 1}$.

CONTRAST BETWEEN ENGINEERING FRESNEL REFLECTION FUNCTIONS

Consider a dielectric in air, its relative refractive index is $n_r$, and $n_r > 1$. Selecting $n_r$ = 1.45 and 1.75, using Formulae (3), (6), (4a) and (7) to calculate the precision FRF, the Schlick EFRF, the S-polarisation EFRF and the simplified EFRF. The three latter EFRFs can be compared with the precision one (3).

Simulated plots are depicted in Figure 1, Figure 2 and Figure 3.

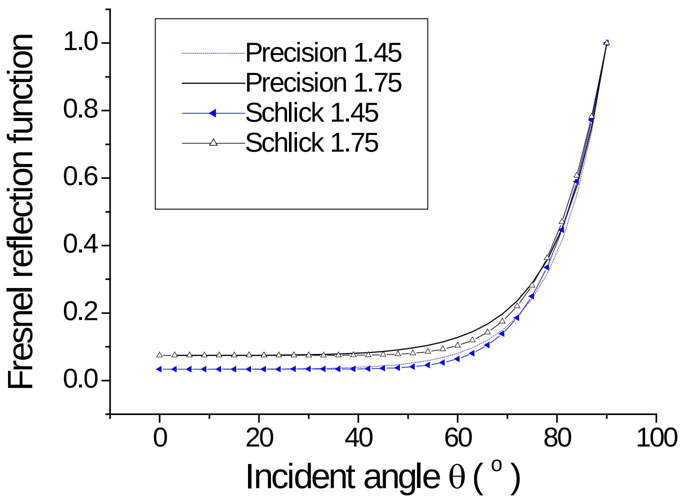


Figure 1: Comparison of the precision FRF and the Schlick EFRF ( $n_r$ = 1.45 and 1.75).

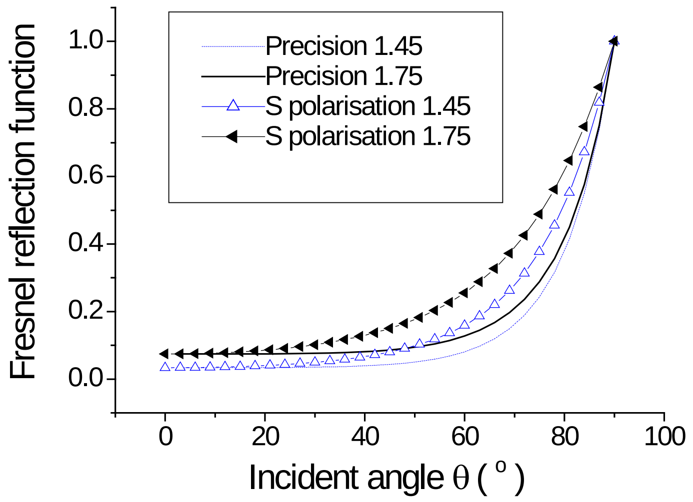


Figure 2: Comparison of the precision FRF and the S-polarisation EFRF ( $n_r$ = 1.45 and 1.75).

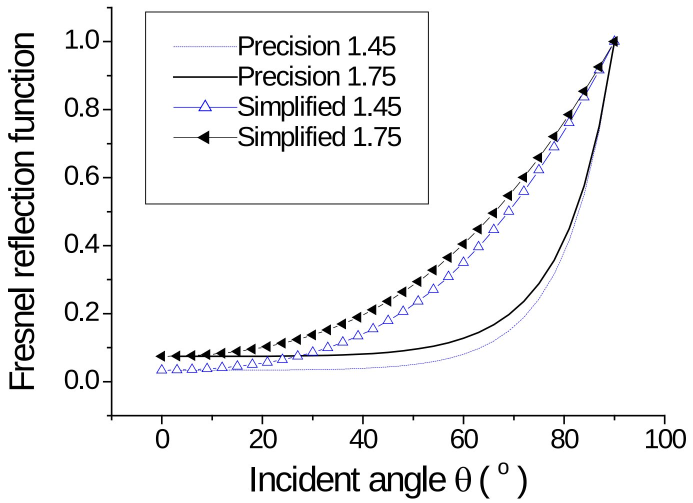


Figure 3: Comparison of the precision FRF and the simplified EFRF ( $n_r$ = 1.45 and 1.75).

The results reveal the following:

- All the EFRFs tend to total internal reflection as for the precision ERF when in grazing incidence, meaning there is no transmitted wave in the dielectric.
- Of all the EFRFs, the one with its curve closest to the precision FRF's is the most accurate.
- The Schlick EFRF was closest to the precision FRF, followed by the S-polarisation EFRF, while the simplified EFRF had the biggest divergence.

## IMPACTS ON ENGINEERING AND TECHNOLOGY EDUCATION

Teaching about polarised electromagnetic waves is difficult in engineering courses. The research on FRFs reveals that electromagnetic waves with two polarisations present different reflection phenomena. Even for a random incident electromagnetic wave, the polarisation is important in determining the reflected wave. Through numerical simulations of EFRFs at media interfaces, the reflection of electromagnetic waves has been determined. Since the reflection function mirrors energy reflectivity, no reflection phase shift occurred.

## CONCLUSIONS

Precision FRFs cannot be easily calculated at many interfaces; however, the EFRFs may be used. The Schlick EFRF is most accurate of the EFRFs and so is more useful in engineering.

The authors are of the view that results of this research should improve understanding of FRFs and their application by university engineering students.

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