Context-based problems and how engineering students view and learn mathematics

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ABSTRACT: Engineering students' beliefs about mathematics and its learning, particularly in relation to context-based problem solving, remain relatively unresearched. The aim of the study described in this article was to explore students' beliefs on the nature of mathematics and of learning mathematics. As well, the students' performances on context-based mathematics problems were investigated. The research subjects were 80 college students studying first year mechanical engineering. Data were collected through students' responses to a mathematics-related belief questionnaire and students' performances on two mathematical problems with different levels of context. The questionnaire consisted of 17 incomplete statements, each of which had three options examining the nature of mathematics and learning mathematics. Results showed that the students tend to hold beliefs about the nature of mathematics that are instrumentalist and learning mathematics as an active construction of understanding. Also, the authors found that the students' performances on the problem with camouflage context were better than those with a clear or authentic context.

INTRODUCTION

There is no doubt that the students in engineering programmes need to be well prepared in mathematics to successfully complete their professional courses. Engineering students need to understand mathematics, as it is one of the foundations of engineering [1]. Success in engineering depends on students' ability in mathematics, as well as the engineering concepts, as they can not be separated from each other [2]. Thus, engineering students need to understand mathematics because, as said, it is one of the foundations of engineering [1]. To conclude, success in engineering depends heavily on the students' ability in mathematics, as well as in engineering [2].

Students' understanding of mathematics is arguably influenced by their view of mathematics. Meanwhile, students' beliefs and understanding of mathematics may impact on their achievements in mathematics [2]. However, beliefs about mathematics in general may be different from beliefs about mathematics in engineering [2].

Regarding mathematics learning, studies have found that engineering students' views on learning mathematics range from traditional to constructivist. The traditional belief deems that learning through a real world context used by teachers to relate to mathematical concepts may not be interesting and even create confusion [3]. Also, students view mathematical procedures as more important than concepts [3].

Many attempts have been made to discuss beliefs about mathematics learning by involving primary and secondary mathematics students. However, these attempts do not relate to tertiary students, particularly those studying engineering mathematics. Therefore, the aim of this study was to clarify the beliefs held by tertiary engineering students regarding the nature of mathematics and of learning mathematics.

The beliefs related to the nature of mathematics and mathematics learning have been studied by scholars. Op't Eynde et al included beliefs about mathematics education, mathematics as a subject, mathematical learning and problem-solving, and mathematics teaching [4]. Ernest suggests that mathematics-related beliefs include views on the nature of mathematics, mathematics teaching and mathematics learning [5]. The authors of this article have restricted the arguments to the philosophical mathematics-related beliefs summarised by Beswick [6]. Table 1 is a summary.

Beswick [6] based on Ernest [5] indicates a theoretical consistency between philosophical views of the nature of mathematics, mathematics learning and mathematics teaching. For example, an instrumentalist's belief about the nature of mathematics inclines to the view that mathematics mainly concerns skill mastery through the passive reception of knowledge. Despite a consistency among beliefs in each row of Table 1, there is no guarantee of consistency among the beliefs of individuals [6]. For instance, it is possible for a Platonist to show beliefs about skill mastery of procedures rather than an active understanding of the concepts.

Table 1: Beliefs about mathematics, mathematics teaching and mathematics learning (Beswick) (adapted from Beswick [6], based on Ernest [5]).

Beliefs about the nature of mathematics	Beliefs about mathematics learning
Instrumentalist	Skill mastery, passive reception of knowledge
Platonist	Active construction of understanding
Problem-solving	Autonomous exploration of learner's own interests

Mathematical models are encountered by engineering students in almost all engineering courses. The models may be physical, block diagrams, flowcharts, statistical or a set of mathematical equations and formulae. Some models may have characteristics that cause difficulties for engineering students. Scanlan believes that engineering students need to be competent in dealing with models not only in relating them to the corresponding physical phenomena, but also in their mathematical analysis [7]. Thus, Scanlan opined that students must be sufficiently competent in mathematics to analyse a model and to understand its impact [7].

Context-based problems promote an engineering student's mathematical modelling competency, viz. the problem is situated in a real-world setting, which needs to be modelled mathematically [8]. Context-based problems correspond to workplace problems, where students translate engineering problems into equations, solve the equations and check the solution to confirm that it solves the original problem [9].

Mathematical methods for solving quantitative engineering problems constitute a significant portion of an undergraduate engineering course. Engineering knowledge, for the undergraduate with little real-world engineering experience, is largely mathematical comprising theories, formulae and models; numerical values for key quantities; and mathematical procedures for analysis and design [10].

In conducting this research, an investigation was made of the beliefs held by engineering students regarding the nature of mathematics and of learning mathematics. Particular focus was on context-based mathematically expressed problems in accordance with the philosophical beliefs stated by Beswick [6]. Also, the authors studied the pattern of performance of the engineering students' strategies in solving context-based mathematics problems.

RESEARCH METHOD

This was a descriptive explorative study, the aim of which was to determine engineering students' beliefs about the nature of mathematics and of learning mathematics, as well as their performance on context-based mathematics problems. The study subjects were 80 engineering students studying mechanical engineering at Universitas Negeri Surabaya, Indonesia. Data were collected using a mathematics-related beliefs questionnaire and two context-based mathematics tasks. The questionnaire consisted of 17 incomplete statements each with three options for completion indicating a philosophical belief category, as summarised by Beswick [6].

For example, the item examining belief about the *definition of mathematics* had an option describing an instrumentalist view; an option describing a Platonist view; and an option describing a problem-solving view. The item examining belief about the *goal of solving mathematics problems* had an option describing a skill mastery view; an option describing an active construction of understanding view; and an option describing exploration by the learner. The context-based tasks had the same level of difficulty, but with different levels of context, i.e. either a clear (authentic) context or where the context is not obvious or camouflaged.

The students' responses on the questionnaire were scored with a 1) instrumentalist; 2) Platonist or 3) problem-solving; for beliefs about the nature of mathematics, and either 1) skill mastery; 2) active construction of understanding; and 3) autonomous. The incomplete statements and options are illustrated in Table 2.

The set of items includes beliefs regarding mathematics learning. These examine beliefs about the problem-solving processes, i.e. understand the problem, devise a plan, carry out the plan and reflect on the solution.

Table 2: Questionnaire: beliefs items and optional answers.

No.	Beliefs item
1	*(How mathematical concepts and procedures are employed in engineering problems)* About mathematics and engineering science, I agree that ... a. Engineering science will be difficult without considering correct recognised mathematical procedures and concepts (1) b. Engineering science is one of the applications of mathematics where the procedures are very varied and used as a tool to solve engineering problems; therefore, it does not matter if the procedures vary (2) c. Mathematics and engineering can develop concurrently, regardless of whether the mathematical procedures used to solve engineering problems vary (3)

2	*(Confidence level of solving applications or context-based problems)* I will be more confident to start learning to solve mathematics application problems in engineering when ... a. I understand the basic concepts of mathematics associated with the application (1) b. The important thing is that the problem is interesting and challenging rather than the basic concepts related to the application (3) c. My lecturer has determined, which one should be first studied: the concept or the application (2)
3	*(Goal of solving mathematics problems)* My goal when solving a mathematics problem is to ... a. Find the answer to the problem (1) b. Find a new strategy that I might never have attempted before (3) c. Understand more deeply the problem and related lecture topics (2)
4	*(Beliefs in the problem-solving processes: understand the problem)* When solving a mathematical problem, I will be able to understand the meaning of the mathematical problem well if ... a. All the information needed to solve the problem is presented completely and clearly (1) b. The necessary information, even though not all, is presented explicitly (2) c. I can explore the relationship between the information, although I have not yet a complete understanding of the relationships among the information (3)

The score of level of philosophical beliefs varied from 1.00 (instrumentalist) to 3.00 (problem-solving). The scores were categorised as shown in Table 3 following Siswono et al [11].

Table 3: Category level of belief understanding.

Score (S)	Belief of mathematical problem-solving
$1.00 \leq S < 1.67$	As instrumentalist view/skill mastery
$1.67 \leq S \leq 2.33$	As Platonist view/active construction of understanding
$2.33 < S \leq 3.00$	As problem-solving view/learner interest

Regarding the context-based tasks, the pattern of strategies was studied, particularly the students' performance on the context-based mathematics problems.

RESULTS

Engineering Students' Beliefs about Mathematics and of Mathematics Learning

The following table shows the engineering students' beliefs toward the nature of mathematics and of mathematics learning particularly when using context-based problems.

Table 4 shows that of the four items (1 to 4) examining the students' view of the nature of mathematics, no item showed a *problem-solving* view. Item 2 on the real-life application of mathematics was Platonist (1.89). The average of the four items 1.66 means *instrumentalist*. This is primarily seen from the students' response to item 3, which defines mathematics as problem-solving. Instead of regarding mathematics as a discipline that can change and evolve, more than a half (58.75%) selected the option, which describes mathematics as a discipline of calculation, numbers and formulae.

Table 4: Engineering students' beliefs about the nature of mathematics and learning mathematics.

No.	Beliefs items	Mean	SD	Interpretation
1	Engineering students' perspectives of mathematics	1.63	0.80	instrumentalist
2	Real-life application of mathematics	1.89	0.53	Platonist
3	Definition of mathematics	1.46	0.59	instrumentalist
4	How mathematical concepts and procedures are employed in solving engineering problems	1.66	0.62	instrumentalist
5	How to learn mathematics through solving context-based mathematics problems	1.31	0.56	skill mastery
6	Success in employing mathematical concepts/procedures when solving a context-based problem	1.51	0.83	skill mastery

7	Strategies that should be learned to solve a context-based problem	1.75	0.86	active construction of understanding
8	Using mathematical formulae	1.93	0.65	active construction of understanding
9	Using a calculator in solving a context-based problem	2.06	0.64	active construction of understanding
10	What motivates engineering students to learn mathematics	2.01	0.58	active construction of understanding
11	The level of confidence in solving a context-based problem based on the extent to which mathematics concepts and procedures have been mastered	1.23	0.59	skill mastery
12	Main goal of solving context-based problems	1.94	0.54	active construction of understanding
13	Dealing with various solutions to context-based problems	2.88	0.40	exploration of learner's own interests
14	How to understand a context-based problem	1.51	0.71	skill mastery
15	How to devise a plan for strategies when solving a context-based problem	2.14	0.88	active construction of understanding
16	How to carry out the plan when solving a context-based problem	2.00	0.83	active construction of understanding
17	How to look back on the strategies and solution when solving a context-based problem	1.45	0.76	skill mastery

Regarding beliefs about mathematics learning, particularly related to context-based problems as shown by items 5-17, Table 2 indicates that five out of the 13 items show an instrumentalist view, seven items show a Platonist view and only one item shows a problem-solving view.

The average mean score of beliefs about mathematics learning was 1.82 (active construction of understanding). The active construction of understanding view is indicated by item 10. In responding to this item, students tended to say that they are more motivated in learning mathematics when there was a clear direction about what to do and what to solve related to particular topics being taught by their lecturer, instead of believing that it is more important to have a reward/appreciaton, such as gained by skill mastery or being challenged by an interesting problem (exploration).

Only item 13 had a score indicating learner exploration. With a relatively high score of 2.88, most students believe that when they have their own and another peer's solution to a context-based problem, they choose to consider both. They may accept the peer's answer, if the peer is viewed as superior on the task (skill mastery) or accept both solutions, as long as the solutions are the same even though the solutions are arrived at through different methods (active construction of understanding). Overall, the suggestions in Table 2 are that the engineering students in this study viewed both the nature of mathematics and mathematics learning as traditional and instrumentalist.

Students' Performance on Context-based Mathematics Problems

Below are examples of students' responses to the context-based problem dealing with the number of cars in a traffic jam. (See Appendix, Problem 1).

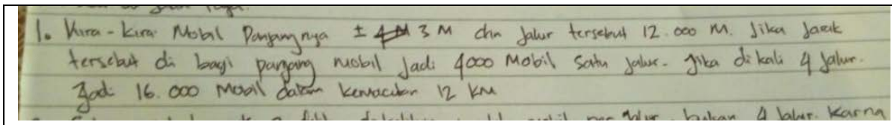
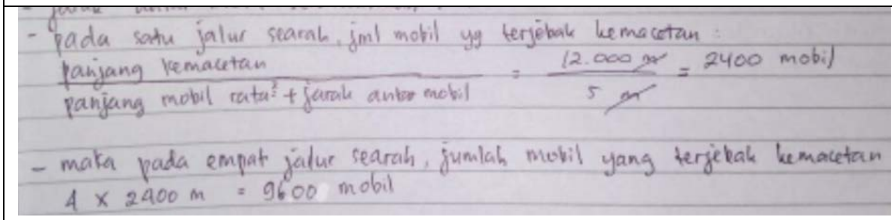
 1. Kira-kira Mobil Panjangnya ± 3 M dan Jalur tersebut 12.000 M. Jika Jarak tersebut di bagi panjang mobil Jadi 4000 Mobil Satu Jalur. Jika di kali 4 Jalur. Jadi 16.000 Mobil dalam kemacetan 12 Km a)	Assume that the length of a car is 3 metres, the length of a road is 12,000 m. If it is divided by the length of a car, 4,000 cars can fit one way. So, for four ways, there will be 16,000 cars
 - pada satu jalur searah, jml mobil yg terjebak kemacetan: panjang kemacetan / panjang mobil rata² + jarak antar mobil = 12.000 m / 5 m = 2400 mobil - maka pada empat jalur searah, jumlah mobil yang terjebak kemacetan 4 x 2400 m = 9600 mobil b)	The number of cars can be calculated by the following: Length of traffic jam (the length of a car + length space between two cars) = 12,000 m/5 m = 2,400 cars. So, for four ways, there would be 4 x 2,400 = 9,600 cars.

Figure 1 a) and b): Examples of students' responses to a context-based problem.

In Figure 1 a) and b) can be seen a sampling of students' responses to the context-based problem. Most students (61 out of 80) gave responses to this problem similar to Figure 1a), while the minority responded as shown in Figure 1b). This means that most students failed to interpret crucial information i.e. the length of space between two cars that needs to be included in the calculation to find the number of cars in the traffic jam. The problem had *authentic* context in that the actual context of the situation needed to be understood to solve the problem.

Compared to problem 1, problem 2, the gear problem (see Appendix, Problem 2) was more successfully performed by almost all students (70 out of 80), who achieved a full score using a variety of methods, such as finding the least common multiple, solving the equation $15x = 20y$ and using gear models. This problem had *camouflaged* context or required no context information i.e. the problem just involved mathematical terms, shapes and data [13].

The students performed better on problem 2 (camouflaged context) rather than problem 1 (authentic context). With the same level of problem difficulty, the students presented more fluent strategies for the problem with camouflaged context than the problem with authentic context. This is because Problem 2 was not about a real-world authentic situation as Problem 1 was. Therefore, when solving Problem 1, many students ignored some hidden information, which is crucial to finding a solution to the problem. This indicates the weaknesses of students when dealing with context-based problems in engineering where the problem requires a good knowledge and sense of an authentic situation [9].

CONCLUSIONS

The conclusions of this study are that students tend to hold beliefs about the nature of mathematics that are instrumental and learn mathematics as a constructive understanding of active comprehension. The student strategy in solving problems is to use trial-and-error, as opposed to a systemic strategy. Students' performance on issues with a camouflaged context was better than that with an authentic context.

The authors agree with the view of Scanlan stating that an effective mathematics course for engineering students must involve continuous, deep and informed dialogue between the mathematics and engineering departements [7]. This bridges the gap between educators in engineering and mathematics about the skills needed by engineers in relation to mathematics.

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APPENDIX

Problem 1

An accident caused a traffic jam along a road of 12 km, which consists of four ways. Estimate the number of cars trapped in the traffic jam.

Problem 2 (from Guberman and Leikin [12]).

Two gears, one with 15 teeth and the other with 20 teeth, fit together as shown in the figure. Each gear has a marked tooth as indicated in the figure. After how many rotations of the gears will the marked teeth be together again in the same formation for the first time?